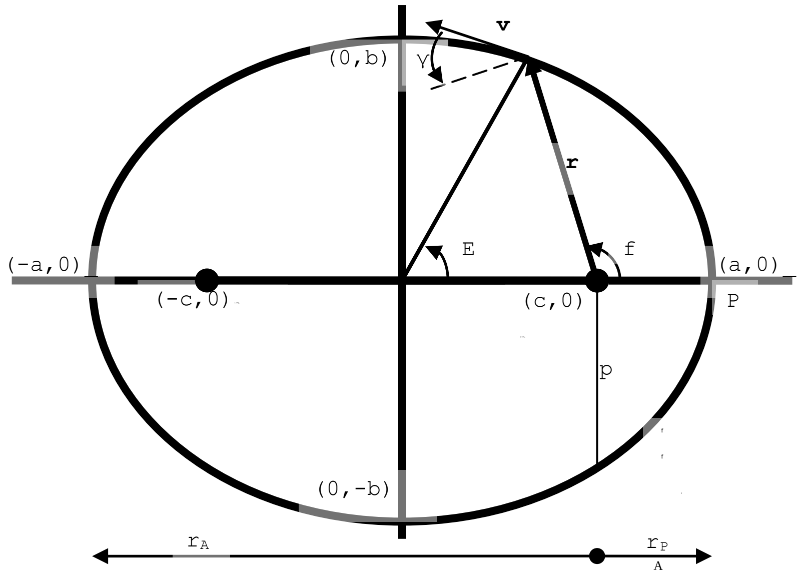
# EllipseEquations2016.doc Ellipse and Orbital Equations




A = aphelion point $A = (-a,0)$

a = semi-major axis
$$a = [r_A + r_P] / 2$$
$$= r/[2 - v^2 r/\mu]$$
$$= r_A/(1+e) \quad = r_P/(1-e)$$
$$= \sqrt{r_A r_P/(1-e^2)}$$
$$a^2 = r_A r_P/(1-e^2)$$

b = semi-minor axis
$$b = \sqrt{a^2-c^2}$$
$$= a\sqrt{1-e^2}$$
$$= p/\sqrt{1-e^2}$$

c = focus distance
$$c = \sqrt{a^2-b^2}$$

E = eccentric anomaly
$$\cos(E) = (e + \cos(f)) / (1 + e\cos(f))$$
$$= e + (r/a)\cos(f)$$
$$= e + \cos(f)\ (1 - e^2)/(1 + e\cos(f))$$
$$E - e\sin(E) = 2\pi\Delta t/P \quad \Delta t \text{ since pericenter}$$
$$\sin(E) = \sin(f)\sqrt{1-e^2}/[1+e\cos(f)]$$

e = eccentricity
$$e = c/a$$
$$= 1 - r_P/a$$
$$= r_A/a - 1$$
$$= [r_A - r_P]/[r_A + r_P]$$
$$= \sqrt{1 - (v^2 r/\mu)[2 - (v^2 r/\mu)]\cos^2(\gamma)}$$
$$= [v_P^2 - v_{Circ}^2] / v_{Circ}^2 = [v_{Circ}^2 - v_A^2]/ v_{Circ}^2$$

f = true anomaly
$$\cos(f) = [2ar_P - ar - r_P^2] / [ar - r_P r]$$
$$= [a(1-e^2) - r]/(er)$$
$$= [(v^2 r/\mu)\cos^2(\gamma) - 1]/\sqrt{1 - (v^2 r/\mu)[2 - v^2 r/\mu]\cos^2(\gamma)}$$
$$= [(v^2 r/\mu)\cos^2(\gamma) - 1] / e$$

$= [r_P(1+e) - r]/(er)$
$= a(\cos(E) - e)/r$
$= [\cos(E) - e]/[1 - e\cos(E)]$
$\tan(f/2) = \sqrt{(1+e)/(1-e)}\tan(E/2)$

G = Newton's constant $= 6.6726\text{E-}11\ m^3/s^2kg$

H = specific ang mom $= r_1 v_1 \cos(\gamma_1) = r_2 v_2 \cos(\gamma_2)$
$= L/m$

K = kinetic energy $K = (1/2)mv^2$

£ = spiral constant £ $\equiv v^2 r/\mu$
$= \sqrt{1 - \beta\cos^3(\alpha) + \beta\cos^2(\alpha)\sin(\alpha)\tan(\gamma)}$

L = angular momentum $= \sqrt{GMm^2}\sqrt{b^2/a}$
$= mrv\sin(\gamma)$

M = central mass $M = 1.989\text{E}30$ for Sol

m = orbital mass $m =$

P = perihelion point $= (a,0)$

*P* = period $P^2 = (4\pi^2/\mu)a^3$
$P = 2\pi\sqrt{a^3/\mu}$

p = semi-latus rectum $p = a(1-e^2) = \mu r$
$= r_P(1+e) = r_A(1-e)$

r = radius from focus $r = [a(1-e^2)]/[1+e\cos(f)]$
$= [p/[1+e\cos(f)]$
$= (2r_A r_P)/[(r_A + r_P) - (r_A - r_P)\cos(f)]$
$= a(\cos(E) - e)/\cos(f)$
$= a(1-e\cos(E))$
$= 2a/[1+(av^2/\mu)]$

$r_A$ = aphelion distance $r_A = a(1 + e)$
$= 2a - r_P$
$= r_P(1+e)/(1-e)$
$= r(1+e)/[2 - (v^2 r/\mu)]$

$r_P$ = perihelion distance $r_P = a(1 - e)$
$= 2a - r_A$
$= r_A(1-e)/(1+e)$
$= r(1-e)/[2 - (v^2 r/\mu)]$
$= r[1-\sqrt{1-[2-(v^2r/\mu)](v^2r/\mu)\cos^2(\gamma)}]/[2-(v^2r/\mu)]$

$\Delta t$ = time from P $\Delta t = (E - e\sin(E))(P/2\pi)$
$\Delta t = (E - e\sin(E))\sqrt{(a^3/\mu)}$

U = Potential energy $U(r) = -m\mu/r$
$U(\infty) \equiv 0$
$U(r_P)/m = -\mu/r_P = -(\mu/a)/(1-e)$
$U(r_A)/m = -\mu/r_A = -(\mu/a)/(1+e)$

v = velocity $v^2 = [2\mu/r - \mu/a]$ vis viva
$v_A = \sqrt{(\mu/a)}\sqrt{(1-e)/(1+e)}$
$v_A = v[1 - \sqrt{1-[2-(v^2r/\mu)](v^2r/\mu)\cos^2(\gamma)}]/[(v^2r/\mu)\cos(\gamma)]$
$v_A = v[1 - e]/[(v^2r/\mu)\cos(\gamma)]$
$v_A^2 = [2\mu r_P] / [r_A(r_A + r_P)]$
$v_A^2 = (\mu/r_A)(1-e)$
$v_P = \sqrt{(\mu/a)}\sqrt{(1+e)/(1-e)}$
$v_P = v[1 + \sqrt{1-[2-(v^2r/\mu)](v^2r/\mu)\cos^2(\gamma)}]/[(v^2r/\mu)\cos(\gamma)]$

$$v_P = v[1 + e]/[(v^2 r/\mu)\cos(\gamma)]$$
$$v_P = \mu[1 + e]/[(vr)\cos(\gamma)]$$
$$v_P^2 = [2\mu r_A] / [r_P(r_A + r_P)]$$
$$v_P^2 = (\mu/r_P)(1+e)$$
$$v_{CircA} = \sqrt{\{(\mu/a)\}}\sqrt{\{1/(1+e)\}}$$
$$v_{CircP} = \sqrt{\{(\mu/a)\}}\sqrt{\{1(1-e)\}}$$
$$v_{Circ}^2 = \mu/r$$
$$v_{Parab}^2 = 2\mu/r$$
$$[vr\cos(\gamma)] = [v_P r_P] = [v_A r_A]$$
$$(v^2 r/\mu) = 2 - r/a$$
$$v_A v_P = \mu/a$$

x = co-ordinate $\quad x = c + r\cos(f)$

$x = ae + r\cos(f)$

$1 = (x/a)^2 + (y/b)^2$

y = co-ordinate $\quad y = r\sin(f)$

$= b\sqrt{\{1 - (x/a)^2\}}$

Useful for logarithmic spiral

$\alpha$ = pitch angle of solar sail

$\beta$ = lightness number, relative acceleration of face-on solar sail.

$\gamma$ = flight angle

$\cos(\gamma) = r_p v_p/rv = r_a v_a/rv$

$\tan(\gamma) = e\sin(f)/(1+e\cos(f))$

$\cos^2(\gamma) = [1 + e\cos(f)]^2 / [1 + 2e\cos(f) + e^2]$

$= [a^2(1-e^2)]/[r(2a-r)]$

$= (1-e^2)/[(v^2 r/\mu)[2-(v^2 r/\mu)]]$

$\sin^2(\gamma) = (e\sin(f))^2/[1 + 2e\cos(f) + e^2]$

max $\cos(\gamma)$ at $\quad \cos f = -e$

max $\cos(\gamma)$t $\quad r = a$

max $\cos^2(\gamma) = 1-e^2$

max $\gamma$ at $\quad (x,y) = (0,b)$

$\tan(f) = r\sin(f)/(x-c)$

$\tan(\gamma) = e\sin(f)/(1+e\cos(f))$ eq.4.64

spiral

$\tan(\gamma)/[2+\tan^2(\gamma)] = [\beta\cos^2(\alpha)\ \sin(\alpha)]/[1 - \beta\cos^3(\alpha)]$ Eq.4.45

$\mu$ = gravitational parameter $\quad \mu = (M+m)G$

$\mu$~ = reduced gravitational parameter $\quad \mu\sim = \mu(1-\beta) = (M+m)G(1-\beta)$ $\quad$ to be used with a solar sail face on to Sol with lightness number $\beta$