Study notes for Chapter Two of *Solar Sailing: Technology, Dynamics, and Mission Applications* by Colin R. McInnes   last edit 2016 March 11 MBM

pXXX.YY means page XXX, line YY. Making a guide is useful.

p33.06 Change to "two metrics: the solar sail loading, and characteristic acceleration or lightness number.", as characteristic acceleration equals (5.96)*(lightness number). $a_0 = (5.96)*(\beta)$.

p34.04 Olbers proposed charge on Sol 1812, but recent estimates show this is only ~+80 coulombs, see MBM 2000.

p36.04 P ≡ force/area. force = dp/dt. Put eqs. 2.6,2.7 into eq. 2.8. to get eq. 2.9.

p38.39 eq.2.19a $\mathbf{u_i} \bullet \mathbf{n} = u_i\ n \cos\alpha$. $[A(\mathbf{u_i}\bullet\mathbf{n})]$ is projected area of sail perpendicular to u_i.
    eq.2.19b has $\mathbf{u_i}\bullet\mathbf{n}$ because with a perfect reflector there is no energy absorbed.

p39.02 Redraw fig.2.3 with sail at an angle of 35° or so instead of 45°. It shows that $\mathbf{u_i}$ and **n** are not perpendicular. Locate sol at bottom of diagram.

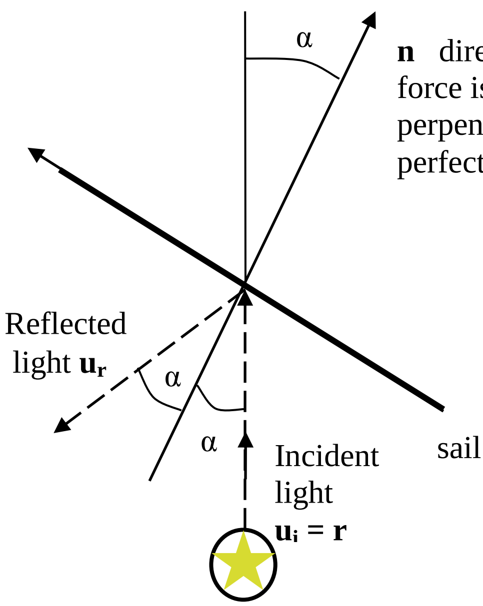


Fig.2.3

p39.15 prove the identity.
    $f_i + f_r = PA(\mathbf{u_i}\bullet\mathbf{n})(\mathbf{u_i} - \mathbf{u_r})$, thus eq.2.20.

p38.24 sail loading $\sigma = m/A$. Define another figure of merit of sailcraft with zero payload: σ_f = (sail mass + structure mass)/(sail area)

p38.29 divide eq.2.21 by *m* to get eq.2.22. eq.2.22 add parentheses to $(\cos^2\alpha)\mathbf{n}$

p40.05 β AKA lightness number is defined with α = 0, and is the absolute value of the ratio of (photon acceleration) to (gravitational acceleration).
    In the extreme case: Blackbody sail does not reflect but does emit IR, perhaps unequal front and back.

p40.25 Add to text: $a_0 = 9.12/\sigma^* = 5.96\beta$. From eq.2.23 at critical load.

p41.38 Hwence the relation $d^3p = p^2 dp d\Omega$? I'm lost.

p42.16 Hwat is "angular moment"? Section 2.4.2 loses me. Torque?

p43.36 "uniform brightness of solar disc" per solid angle? Fig. 2.5 says use center as distance r.

R_{Sol} = 6.9E8 m, a_{Terra} = au = 1.5E11 m, $a_{Mercury}$ = 5.79E10 m.

p44.14 Here Fig. 2.5b looks down at plane containing **n** and u. $\lambda = 90$ if $\theta = \theta_0$.

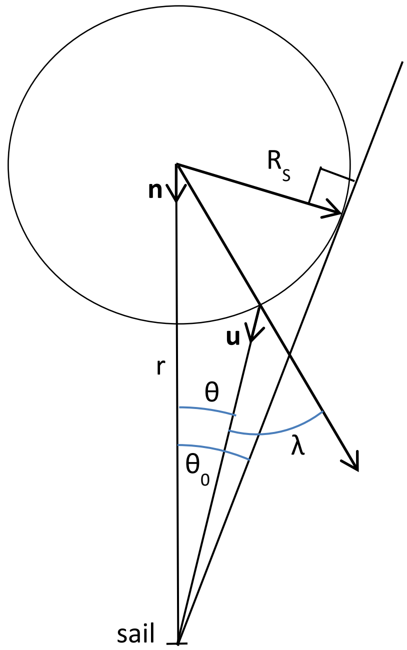


Fig.2.5b

Looking down on cross section thru center of Sol

$\sin\theta_0 = R_s/r$

$\sin\lambda = \sin\theta / \sin\theta_0$

ϕ is rotation about r

p44.26 To go from eq.2.38 to eq.2.39, integrate, then substitute $(1-\sin^2\theta_0)^{1/2}$ for the cosine. Using $\sin\theta_0 = R_s/r$, eq. 2.39 can be written $P(r) = (4\pi/3c) I_0\{1 - \cos^3\theta_0\}$.

Expand eq. 2.39 with binomial theorem.

$(1-x)^{3/2} = 1 - (3/2[x] + (3/8)[x]^2 + (1/16)[x]^3 + \ldots$ With $x = [(R_S/r)^2]$

$P(r) = (4\pi/3c) I_0 \left(1 - \{1 - (3/2)[(R_S/r)^2] + (3/8)[(R_S/r)^2]^2 + (1/16)[(R_S/r)^2])^3 \ldots\}\right)$

$F(r) = (4\pi/3c) I_0 \left((3/2)[(R_S/r)^2] - (3/8)[(R_S/r)^2]^2 - (1/16)[(R_S/r)^2])^3 \ldots\right)$

$F(r) = (2\pi/c) I_0 \left((1)[(R_S/r)^2] - (1/4)[(R_S/r)^2]^2 - (1/24)[(R_S/r)^2])^3 \ldots\right)$

$F(r) = (2\pi/c) I_0)[(R_S/r)^2 \left((1) - (1/4)(R_S/r)^2 - (1/24)(R_S/r)^3 \ldots\right)$

p45.20 Fig. 2.6 has the vertical axis at r = $1.0R_{Sol}$, not r = $0R_{Sol}$, which would be more clear. Label horizontal axis in units of R_S, starting explicitly at 1. $F(1R_S) = 2/3$. Mercury is at $a_{Mercury} = 83.9R_S$, and the deviation from $1/r^2$ is negligible. a_{Terra} = 1au = $217R_S$.

At $1R_S$ Sol looks like an infinite plane. The energy flux thru a surface near and parallel to the solar surface is ~constant with r, but the momentum flux is not proportional to energy flux because in general photons are not moving perpendicularly thru the surface. Thus the factor of 2/3 at $1R_S$.

p45.30 Eq.2.44a should be $P(r) = [P^*(r)][F(r)]$

Expand eq. 2.44b with binomial theorem, as at eq.2.39. $F \to 1$ as $r \to \infty$.

P45.45 Now look at Terra as an extended source of reflected sunlight and emitted infrared, compare this intensity with solar intensity out to 100 terrestrial radii. See notes for p54. Also look at $r \to R_S$.

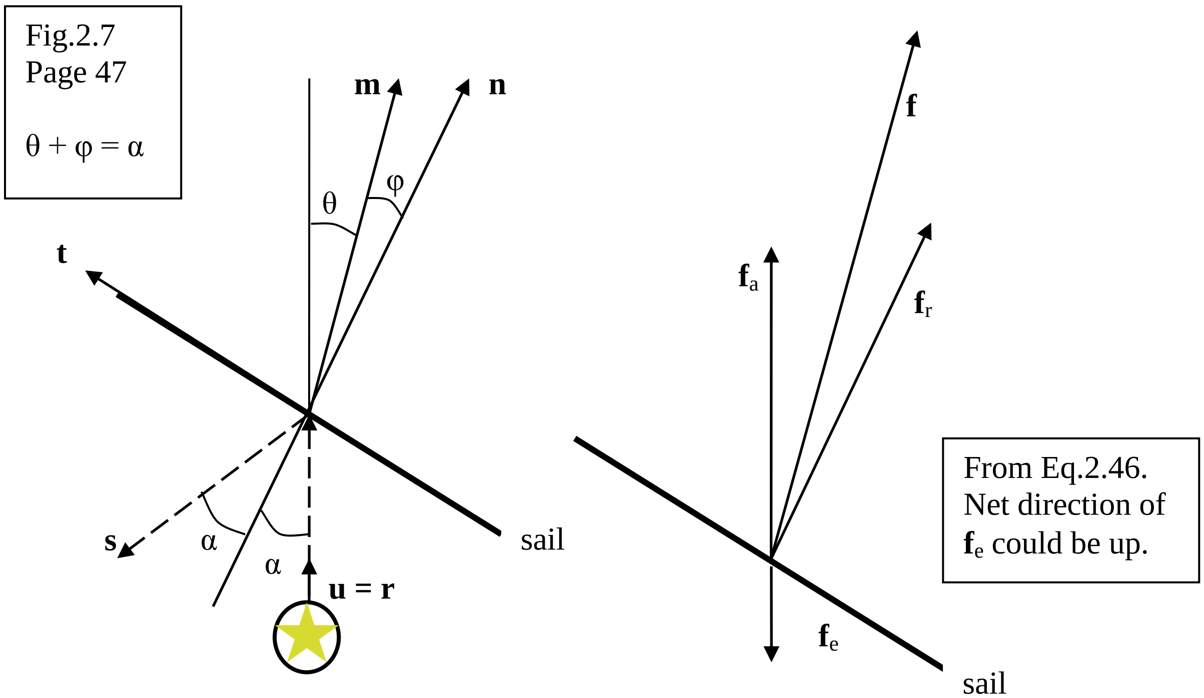
p46.06 Need to learn more about limb darkening. Absorption and scattering? From eq. 2.45, at very large r, max $\lambda = 90$, $I = I_0/2 \neq I_0$. ? Is I_0 measured at large r?

At the limb, $\theta = \theta_0$, and $\lambda = \pi/2$, so $I(\text{limb}) = (I_0/4)(2-3\cos(\pi/2) = I_0(2/4)$.

At the center, $\theta = 0$ and $\lambda = 0$, so $I(\text{center}) = (I_0/4)(2+3\cos 0) = I_0(5/4)$.

I(limb) does equal 0.4 I(center). How does this lead to $F(R_S) = 0.708$? Compare with Fig. 2.6.

p47.04 Redraw fig.2.7 with α = 35° so that **s** does not appear perpendicular to **u**. Angle between **s** and −**n** is also α. Note Sol is at bottom. Add force diagram for eq.2.46



p47.23 The optical force model assumes a flat but imperfectly reflecting sail.
See new drawing of $\mathbf{f}_r$, $\mathbf{f}_a$, $\mathbf{f}_e$. Need to include aberration angle?

p47.37 It is not clear that τ in Eq.2.48 should be zero on front of sail. These sections are not clear. See my note at end of chapter.
Specular = mirror-like, angle of incidence = angle of reflection.

p48.02 (cosα)**n**, (sinα)**t** et cetera makes it clear

p48.21 "uniformly scattered" needs justification. Is aberration significant? Demonstrate yes or no.

p48.44 Same temperature front and back? Yes, for a thin sail.

$\boldsymbol{f}_r = \boldsymbol{f}_{rs} + \boldsymbol{f}_{ru}$ = Õ$rs\boldsymbol{f}_a$ + B_fÕ$r(1-s)PA(\cos\alpha)\mathbf{n}$ =

= PA{[(Õ$rs(\cos^2\alpha)$ + B_fÕ$r(1-s)(\cos\alpha)$]$\mathbf{n}$ − [Õ$rs(\cos\alpha)(\sin\alpha)$]$\mathbf{t}$}

In eq. 2.54 missing an "A" after "T^4A".

Hwy all absorbed? Because then $\tau = 0$?

p49.18 In eq.2.55, the transmission coefficient should also be subtracted. For r~ = 0.88, c = 3E8 m/s, $P = 9.12 \cdot 10^{-6}(r/r_{Terra})^2$ N/m^2, α = 30°, σ~ = $5.67 \cdot 10^{-8}$ W/(m^{2}K^4), ε_f = 0.05, ε_b = 0.55, then by eq.2.55 some sail temperature are:

T(Mercury = 0.387 au) = 486K
T(Venus   = 0.723 au) = 356K
T(Terra   = 1.000 au) = 302K
T(Mars   = 1.524 au) = 245K.

p49.39 "in the direction of the total force" could read "in the direction of the total force (= reflected plus absorbed)" and get rid of the s parameter.

p49.42   "Since a real sail ..." This matters, see fig. 2.10 for the table 2.1 model.

p49.44   "somewhat greater than that due to reflected (re-radiated specularly) photons."? Yes, because all photons are absorbed in this formulation. I guess.

p50.19 Fig.2.9 Vertical scale is exaggerated by a factor of two if you just think of PA. From p13-14 P = 9.12 x $^{10-}$6 N/m2. Multiply by 10^4 m^2 to get 0.0912 N max value for sail facing Sol.

p50.22 Figs. 2.9-2.12 based on table 2.1, imperfect sail.

p50.30 after "biased" change to "biased from normal toward direction of incident radiation. "

p50.37 Add "flat" to read "for a perfectly reflecting and flat solar sail".

p51.19 Fig.2.10 For a perfect sail the centerline angle is always 0, see p50b. Peak at 72.6 °.

p52.20 Fig. 2.11 based on table 2.1, p50.

p53.20 Fig.2.13 Normalized force is f/f0 = {1 +(-2c2 -8c3)sin2 θ + 8c3sin4θ}m, since c1 + c2 + c3 = 1. Note that fig.2.13 and eq.2.60 can imply a negative force for large cone angles, so limited range of good approximation.

p53.22 Table 2.2 Cs should be lower case.
For perfect sail, α = 35° is most efficient. Imperfect, closer to 30°.

p53.37 Note that fig.2.13 and eq.2.60 use cone angle θ and not pitch angle α.
An imperfect sail of mass m at some pitch angle α will experience a radial force $\underline{f_r}$ and transverse force $\underline{f_t}$. Define an effective pitch angle $\underline{\alpha_\alpha}$ = atan($\underline{f_t}$/$\underline{f_r}$).
Find an equivalent (smaller) perfect sail. Let its force at pitch angle zero be $\underline{f}$. We want $\underline{f}$ = $\underline{f_r}/\cos^2(\underline{\alpha_\alpha})\cos(\underline{\alpha_\alpha})$ or equivalently and $\underline{f}$ = $\underline{f_t}/\cos^2(\underline{\alpha_\alpha})\sin(\underline{\alpha_\alpha})$. It will have an equivalent lightness number $\underline{\beta_\alpha}$ = [$\underline{f}$/m] / [GM/(1au)^2]. Using (alternate) Eq.4.45, $\underline{\beta_\alpha}$ and $\underline{\alpha_\alpha}$ lead to a flight angle $\underline{\gamma_\alpha}$, and defines the logarithmic spiral. One also needs to know $\underline{\beta_0}$ the relative acceleration hwen the imperfect sail is face on to Sol.

Example. Set the imperfect sail at 33°. Assume measurements show that $\underline{f_r}$ = √3 N and $\underline{f_t}$ = 1 N. Then $\underline{\alpha_{33}}$ = atan($\underline{f_t}$/$\underline{f_r}$)= atan(1/√3) = 30°.
$\underline{f}$ = $\underline{f_r}/\cos^2(\underline{\alpha_\alpha})\cos(\underline{\alpha_\alpha})$

p54.06 Proton mass m_p = 1.67E-27 kg. In one second protons from a column 1 m^2 in cross section and 7E5 m long will strike 1 m^2 of sail, each with momentum (1.67E-27 kg)*(7E5m/s), thus eq.2.61.

p54.13 Solar wind response 1/10,000 compared to photon wind. But, solar wind collector can be much larger as well as less massive.

p54.15 Confirm magnitude of aberration of solar photons is 1/1000. Harwit p177 says this becomes important if areal density is about 10 g/m^2.
Poynting-Robertson, F/A = $(1/4c^2)\sqrt{(GM_SL_S/R^5)}$ = 1.46E-9 N/m^2 at 1.0 au
= 1.44E-8 N/m^2 at 0.4 au
Reflection Force = 9.12E-6 N/m^2 at 1.0 au

p54.16 Terrashine in LEO is less that 1/1000 Solshine? I get >1%. See below.

Terra as a source of light for a photon sail.
Ls Luminosity Sol 3.8E26W
Lt Average luminosity Terra (reflected, infrared)
Rs Radius Sol = 6.9E8 m
Rt Radius Terra = 6.37E6 m
Ds Sol-Terra distance = 1au = 1.5E11 m
Dt Terra-sail distance
Ws Power density of Sol at Ds Ws = Ls / 4piDs^2 = 1344 W/m^2
Wt Power density of Terra at Dt
Lt = Ws * pi*Rt^2 since radiates hwat it absorbs
Wt = Lt / 4piDt^2

= Ws * pi * Rt^2 / 4pi * Dt^2
= Ws (Rt/2Dt)^2
Ws/Wt = 4(Dt/Rt)^2
If Dt = Rt + 900 km = 7270 km = 1.14 Rt then Wt = .19Ws LEO
Dt = Rt + 1592 km = 7962 km = 1.25 Rt then Wt = .16Ws LEO
Dt = Rt + 6370 km = 12740 km = 2.00 Rt then Wt = .0625Ws Ws/Wt = 16
Dt = 42240 km = 6.63 Rt then Wt = .0057Ws Clarke

The low radii do not treat Terra as an extended source, worst case is about 2/3 these values.

p54.22 Air drag typically starts at 900km

Note. Alternative derivation of force on imperfect sale.

Assume for current sail materials the transparency will be zero, and that sail is flat. The incoming light has a fraction s of specular reflection, a fraction d of diffuse reflection, and a fraction a absorbed, with s+d+a = 1. Then
$f_s = s2PA(\cos^2\alpha)\mathbf{n}$ oriented in the direction **n** perpendicular to the sail.
$f_d = d2PA(\cos\alpha)B\mathbf{n}$ per page 48m.
$f_a = aPA(\cos\alpha)\mathbf{u}$ oriented along the sun-line **u**. If the sail is thin the temperature will be the same front and back. Assume front and back emissivities ε_f and ε_b, with $\varepsilon_f < \varepsilon_b$. Re-radiation front and back will be on average perpendicular to the sail. By symmetry, assume the Lambertian coefficient B is the same front and back. For front emission the force is $f_f = f_a(\varepsilon_f/(\varepsilon_f + \varepsilon_b))B\ \mathbf{n}$ and for back emission it is
$f_b = f_a(\varepsilon_b/(\varepsilon_f + \varepsilon_b))B(-\mathbf{n})$. The net force is
$f_n = f_a((\varepsilon_f - \varepsilon_b)/(\varepsilon_f + \varepsilon_b))B\mathbf{n}$. Then the total force

$f_T = = s2PA(\cos^2\alpha)\mathbf{n} + d2PA(\cos\alpha)B\mathbf{n} + aPA(\cos\alpha)\mathbf{u} + aPA(\cos\alpha)((\varepsilon_f-\varepsilon_b)/(\varepsilon_f+\varepsilon_b))B\mathbf{n}$. The resultant force **f** will be in the direction **m** at angle θ to the sun line. Note that for any pitch angle α a perfect sail may be found with the same response as the imperfect one. - MBM