Study notes for Chapter Three of *Solar Sailing: Technology, Dynamics, And Mission Applications* by Colin R. McInnes

pXXX.YY means page XXX, line YY. Making a guide is useful.

p56.24 Does photon pressure inflate an unsupported sail at all, or does it just turn it into a streamer?

p58.11 "*m* total ~~solar sail~~ sailcraft mass", mass of sail plus support plus payload. With sail assembly loading $\sigma_S = m_S/A$, can also write eq.3.2 as $a_0 = 2\eta PA/(\ m_S + m_P\ )$.

p58.43 Does not increasing sail area mean more than proportional increase in structure? Then m_S is a function of sail area, and eq.3.3 gets more complicated.

p59.17 Fig.3.1 is from eq.3.3.

p59.40 From eq. 3.2, $a_0 = 2\eta P/[\sigma_S + (m_P/A)]$. $\sigma_S = (m_S/A)]$. From eq. 3.4

$$\Delta a_0 = (\partial a_0/\partial\sigma_S)(\Delta\sigma_S) = (2\eta P)(-1)[\sigma_S+(m_P/A)]^{-2}(\ \Delta\sigma_S\ )$$

$$\Delta a_0/a_0 = (2\eta P)(-1)[\sigma_S+(m_P/A)]^{-2}\ \Delta\sigma_S\ \ /\ \{2\eta P/[\sigma_S+(m_P/A)]\} =$$

$$= (-1)[\sigma_S+(m_P/A)]^{-2}\ \Delta\sigma_S\ \ [\sigma_S+(m_P/A)] = (-1)\ \Delta\sigma_S\ /[\sigma_S+(m_P/A)] =$$

$$= (-1)(\Delta\sigma_S\ /\sigma_S)/[\sigma_S/\sigma_S)+(m_P/A)/\sigma_S)] = \ (-1)(\Delta\sigma_S\ /\sigma_S)/(1+(m_P/A)/(m_S/A))] =$$

$$= \ (-1)(\Delta\sigma_S\ /\sigma_S)/(1+(m_P/(m_S))] = \ [-1/(1+(m_P/(m_S))]\ (\Delta\sigma_S\ /\sigma_S) = \text{eq. 3.5a.}$$

p60.01 Tab.3.1. To caption add "155kg total mass, η = .85". From p14 8g/m^2 is typical all up loading if a_0 = 1.0 mm/s^2. 6g/m^2 for sail and structure gives 60kg for 100m x 100m sail. This is not the 1:2 payload:sail ratio used earlier. Payload 55kg => sail 10g/m^2.

Add columns:

Assembly Loading σ_S g/mm^2	Sail plus structure mass in kg	payload mass m_P in kg	total mass m in kg	sailcraft load σ in g/m^2	Λ_1	Λ_2	Λ_3
6	60	95	155	15.5	-0.39	-0.61	0.39
4	40	115	155	15.5	-0.26	-0.74	0.26
2	20	145	155	15.5	-0.13	-0.87	0.13

p60.17 It is always true that $\Lambda_1 = -\Lambda_3$, and that $\Lambda_1 + \Lambda_2 = -1$.

p61.18 1 micron Mylar 1.3g/m^2 for ultralight airplane models. They also use monolayer wing covering on the order of 10^{-3} g/m^3. See F1d Indoor Duration model aircraft, "watercasting" p62m.

p61.34 Table 3.1. Need example of ensile forces for 100m x 100m sail

p61.43 Calculate areal density for a given required tensile strength. I get Mylar barely beats Kapton, but both are well ahead of Lexan. This assumes that thickness is determined by required strength and not minimum thickness attainable. And assumes tensile strength is proportional to thickness.

Kapton(R) (2.82 g/m^2) + Al (0.27 g/m^2) + Cr (~0.1 g/m^2) = 3.2 g/m^2.

p62.41 Is there a reflector alloy with high infrared emissivity, such that it emits more than accepts from Sol? Snow is very reflective in visible, very black in IR, not much of a structural material. Front emissivity is not zero.

p62.44 "high density" in g/m^2, not kg/m2.

p63.01 Aluminium is aluminum in USofA, and can have areal density of 0.25 g/m^2, per Eric Dexler.

p63.35 Sail minimum distance from sun is 0.2 au = 42 R_S unless $\alpha>0$.
Mercury at 84 R_S = 0.4au. $cP = W_T(R_T/r)^2$ hwere I have used subscript T instead of E. Note that r is distance from Sol, r~ is reflectivity.
Thermal emission from back of sail counters some of the photon pressure.

p64.20 Per p63m, draw in the 520K line in fig.3.2.

p65.11 After sail and substrate delaminate, how do they separate? Photon pressure forces them against each other. Evaporation on Sol side and resultant rocket acceleration may work better.
After sail and substrate delaminate, does some structure become redundant?

p66.05 How thick is a few hundred atoms? For aluminum ~ 0.4 μm per layer, but at some point transparency is important.
IR hwiskers on front of sail as per p63b comment?

p66.08 Can you stretch films in orbit to get thinner? Zigzag ripstop.

p66.40 A spar supported sail is under tension. Hwy no ripstop needed?

p67.19 Fig.3.5 Script "l" on diagram and Roman "l" of text should be the same character. Script l ≈ $\lambda_{IR}/4$ is length of radiator. Replace "l" of eqs. with "script l", move "a", "b", "c" out of drawing. Hwat is importance of dimension "c"?

p68.40 Diagonal pattern is an advantage during deployment, see p76m.

p69.27 IKAROS from JAXA is a spinning square sail.

p69.40 How does cross orienting layers eliminate thermal expansion? Non-isotropic?

p73.26 Rectangular sail with two spars $D_R/4 = (8A)^{0.5}/4 = (A/2)^{0.5}$ held apart by single spar twice as long also has area $(D_R/4)*(2\ D_R/4) = [(A/2)^{0.5}][2(A/2)^{0.5}] = A$, and there may be advantages for attaching sail to spar, or fabricating in space.

p73.34 Eq. 3.9 does not signify. The forces determine the mass, hwich is the important parameter. Moment of inertia is a lesser parameter. Forces first, then engineering, then compare. McInnes does acknowledge this in bottom paragraph.

p74.06 same argument as above for Eqs. 3.10 and 3.11, need to know forces and masses first. Also, earlier sail mass and structure mass were assumed to be =~, so the former should not be left out, even tho it is similar for square and disc.

p74.09,.19 Eqs. 3.10 &3.11 confirmed. Eq. 3.12 has no use, as noted in following paragraph.

p74.38 Number of spars is of little use, it is the structural mass that counts.

p75.30 See above. Even looking at area per spar length, if spars must be constant mass/length, the triangular or rectangular sail with spars set at 120° is more efficient. A = 0.1443 * (total spar length)^2 for triangle, 0.125 for square with crossed spars, or rectangle (double square) with spars at short ends and one connecting spar. Again, see paragraph bottom of page 75. McInnes's eq. 3.14 compares sails of different total spar length and different area. Eq.3.14, adding spars adds weight with very little more added area, but spars can be lighter.

p76.13 The central post adds more weight but reduces mass of main spars. Again, hwat are the forces on the sail, and how distributed? Can the mast be used as a support for a steering vane?

p76.39 The articulated boom with payload mass is different than the structural post above, and they may be incompatible if deployed on same side of sail.

p77.40 Eq.3.16. ϕ is rotation CCW about +z, from +x to projection of line O- m_p.

p78.18 Fig. 3.13 is confusing.
The diagonals of the sail are the x and y axes.
z is perpendicular to x and y. The unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ are along the three axes.
S is line from Sol, and makes pitch angle α with **z** axis, hwich is on the
Sol side of the sail. **S** is parallel to unit vector **u, and** is not
The same **s** as fig. 2.7. **S** is usually arriving on the +z side of
the sail, but is shown that way in drawing for simplicity.
m_p is mass of payload, a distance l from origin, and the vector from
origin to m_p is $\mathbf{r}_p$ and is not shown in figure.
Θ is angle between $\mathbf{r}_p$ and the x-y plane, and is projected onto that plane.
ϕ is angle between that projection and the x-axis.
n is direction of all resultant forces **f**, and since this is a perfect sail
the net force direction is same as the –z axis. **m** = **n** in the notation
of fig. 2.7. **n** makes angle α with z.
d is spar length.
δ_1 and δ_2 are rotations about the x-axis, measured from the –z axis to
the vane normals $\mathbf{n}_1$ and $\mathbf{n}_2$.

p78.24 m_S (mass of sailcraft) has its center of mass at origin, symmetry.
$r_C = (m_P r_P + m_S r_S)/(m_P + m_S)$ but $r_S = 0$.
Add [and] to "of radius $[m_P/(m_P + m_S)]l$."
$(\mathbf{S}\cdot\mathbf{n})^2 = \cos^2\alpha$. Put eq. 3.16 into eq. 3.17. e_3 // to –**n,** so $M_z = 0$.

p79.12 No torque on main sail hwen edgewise mean you cannot stop sail in edgewise orientation, but try with vanes.

p79.41 Control vanes, back to fig. 3.13. δs are rotation angles of $\mathbf{n}_1$ and $\mathbf{n}_2$, unit vectors normal to vanes, perpendicular to x-axis with $\delta = 0$ hwen they are parallel to **n** = -z^. Rotation is clockwise hwen seen from origin. Distance of vane from origin is *d.*

p80.43 At elevation of 90°, torque is zero (the back horizontal line). Label azimuth with 0, 90, 180, 270, 360 along with instead of 100, 200, 300.

p81.02 $\mathbf{d}_i$ are distances from corner to sailcraft center of mass, excluding payload.

p81.10 eq. 3.23bc . Could not derive these.

p81.31 Hwy is roll a problem unless it becomes too much? It does mean changing orientation for controlling pitch and yaw.
How about control vanes on the central post. Shadow?
Need always to sense direction to Sol, and know current **v** and desired change. How do vane sizes change with increase in sail size to maintain roll authority? Moment arm gets longer.

p82.41 Graphs are a bit confusing. Move labels to different edges of cube?

p83.39 "in plane chordwise", does this mean bending by stretching one side of the blade? Blades must start at some r>0, but over 7500 meters 2 is almost zero.

p83.41 Change "between" to "among".

p84.31 **Ω** is around axis that is perpendicular to page. x is distance from centerline of blade to blade element.

p84.39 eq.3.24ab Here f is force density, N/m^3, not force. Multiply by hCΔr. This convention propagates thru the chapter.

p85.03 In eqs.3.25ab, the hC (cross sectional area) top and bottom cancel.

p85.16 "small displacements" means pressure is essentially constant. Give example of deriving net P_n

p85.25 $dw/dx = 0$ or $\partial w/\partial x = 0$? Does it make a difference?

p85.27 Using the coning angle (bend along the blade, I think) (really slope across blade) $\vartheta \equiv dw/dr$, Eq. 3.28 becomes $d(\sigma_r\vartheta)/dr + P_n/h = 0$. I integrate this to $(\sigma_r\vartheta) + \int P_n/h\ dr$ = constant assumed to be zero, with limits of integration from r to R so that Eq. 3.29 reads $\vartheta = -(1/\sigma_r)\int P_n/h\ dr$, with the minus sign. Substituting using Eq. 3.26a gives
$\vartheta = -(2/\rho\Omega^2(R^2-r^2))\int P_n/h\ dr = -(2/\rho\Omega^2(R^2-r^2))(P_n/h)(R-r) =$
$-(2/\rho\Omega^2(R+r))(P_n/h) = -(2P_n/(\rho h\Omega^2(R+r))$. I think. The minus sign just changes the direction up or down.

p85.41 In eq. 3.30 $dw/dr \equiv \vartheta(R)$ is 5×10^{-4} so it is indeed small.

p86.21 Fig. 3.18 The dashed line is for a straight blade set at the coning angle of the root. The actual blade flattens out because there is less net torque from the pressure of photons as you go out, and the centripetal force takes over. Think about this. Hwat does the caption mean?

p86.32 To get eq. 3.32, integrate eq. 3.30 from 0 to r. Still missing minus sign? Then $w(R) = w(7500\text{m}) = 2.25\text{m}$, less than chord.

p86.43 How to derive eq. 3.33. Confusion from definition changes of w and θ. Revelation. w is still vertical motion, but the new theta is not italicized. Now there can be progress. ϑ θ

Starting with eq. 3.27, multiply by hx, substitute for σ_x (eq. 3.26b) and for w using $w = \theta x$ (p86b).

Since it is assumed that θ does not depend on x (blade is flat, p86b), $\partial\theta x/\partial x$ is θ. And since x is independent of r, in the first term x can come out of the differential. Take the partial of the second term wrt x, noting that the first part of the second term is a constant. Then integrate from –C/2 to +C/2, substitute for the inertia of the cross section about the central axis (eq. 3.34a) and on the rite side using 3.34b (hwy is this the twisting moment?) to get eq. 3.33.

1 Start with eq. 3.27
2 multiply by hx and rearrange
3 substitute for $w = \theta x$ p86b and $\sigma_x(x) = (\rho\Omega^2/2)((C/2)^2-x^2)$ eq. 3.26a
4 Assuming that θ does not depend on x (blade is flat, p86b), $\partial\theta x/\partial x$ is θ.
5 Take derivative of second term
6 Integrate from –C/2 to C/2 with RHS from eq.3.34b
7 Evaluate limits
8 Substitute for $(C^3h/12)$ using eq. 3.34a to get eq. 3.33.

1 $\quad (\partial/\partial r)[\sigma_r(\partial w/\partial r)] + (\partial/\partial x)[\sigma_x(\partial w/\partial x)] + (P_n/h) = 0$
2 $\quad hx(\partial/\partial r)[\sigma_r(\partial w/\partial r)] + hx(\partial/\partial x)[\sigma_x(\partial w/\partial x)] + hx(P_n/h) = 0$
3 $\quad hx(\partial/\partial r)[\sigma_r(\partial\theta x/\partial r)] + hx(\partial/\partial x)[(\rho\Omega^2/2)((C/2)^2-x^2)(\partial\theta x/\partial x)] + x(P_n) = 0$
4 $\quad hx^2(\partial/\partial r)[\sigma_r(\partial\theta/\partial r)] + hx(\partial/\partial x)[(\rho\Omega^2/2)((C/2)^2-x^2)\theta)] + x(P_n) = 0$
5 $\quad hx^2(\partial/\partial r)[\sigma_r(\partial\theta/\partial r)] + hx\ [(\rho\Omega^2/2)((-2x)\theta)] + x(P_n) = 0$
6 $\quad x^3\text{<lim>}(h/3)(\partial/\partial r)[\sigma_r(\partial\theta/\partial r)] - x^3\text{<lim>}(h\rho\Omega^2\theta/3) + t_\theta = 0$
7 $\quad (C^3h/12)(\partial/\partial r)[\sigma_r(\partial\theta/\partial r)] - (C^3h/12)(\rho\Omega^2\theta) + t_\theta = 0$
8 $\quad (\partial/\partial r)[I\sigma_r(\partial\theta/\partial r)] - (\rho\Omega^2 I\theta) + t_\theta = 0$

p87.21 Fig. 3.19 note that left axis, $\theta(r)/\theta(0)$, does not start at zero. I think this is from solving eq. 3.35 as a first order differential equation in $\partial\theta/\partial r$. If the root twist θ is three degrees, then the tip twist will be a bit more than one degree.

p87.24 I is around central axis.

p87.37 From eq. 3.33 using eq. 3.26b (I think) and $t_\theta = 0$ (photon pressure balances) to get eq.3.35. t_θ in eq.e.34b has wrong units, should be an R before Pn in RHS. Explain in detail. *******

p87.37 Deriving eq. 3.35. In eq. 3.33 set $t_0 = 0$ and use eq. 3.26a to substitute for σ_r.
$(\partial/\partial r)[(\rho/2)\Omega^2(R^2-r^2)(\partial\theta/\partial r)] = \rho\Omega^2\theta$. Cancel,
$(\partial/\partial r)[(R^2-r^2)(\partial\theta/\partial r)] = 2\theta$. Take partials
$(-2r)(\partial\theta/\partial r) + (R^2-r^2)(\partial^2\theta/\partial r^2)] = 2\theta$ so $(R^2-r^2)(\partial^2\theta/\partial r^2) - (2r)(\partial\theta/\partial r) - 2\theta = 0$.
Now solve it. See Wolfram Mathworld Second Order Ordinary Differential Equation.
According to fig. 3.19 a linear approximation is possible.

p88.01 Table 3.4 Change caption to "Required root torque M_0 in Nm for a 7.6µm thick x 1m x 300m heliogyro blade." Label the LH column with θ_0. The table comes from eq. 3.37 and shows nine values of M_0 for different root twists and spins. Recalculating:

Table 3.4. Required root torque M_0 in Nm for a 7.6µm thick x 1m x 300m heliogyro blade.

Blade root Pitch angle θ_0 deg (rad)	Root torque: Ω = 0.2 rpm 0.02094 rad/s	Root torque: Ω = 0.3 rpm 0.03142 rad/s	Root torque: Ω = 0.5 rpm 0.05236 rad/s
10° (.1745)	$.657 \times 10^{-5}$ Nm	1.48×10^{-5} Nm	4.10×10^{-5} Nm
20° (.3491)	1.31×10^{-5} Nm	2.95×10^{-5} Nm	8.21×10^{-5} Nm
30° (.5236)	1.97×10^{-5} Nm	4.43×10^{-5} Nm	12.31×10^{-5} Nm

Note: My torques are 0.526 of the book's numbers. Cannot explain the discrepancy.

p88.12 Suspect I do not understand eqs.3.36 and 3.37. Used these values for table:

ρ_{Kapton} = 1420 kg/m^3	density of Kapton p61
C = 1.00 meters	blade width
h = 7.6×10^{-6} meters	blade thickness
I = 6.333×10^{-7} m^3	$(1/12)C^3h$
R = 300 meters	blade length
θ_0 = in radians	blade twist at root
Ω = in radians/sec	spin rate about hub
$\theta_R = 0.364\ \Theta_0$ radians	blade twist at end, p87b
$\sigma_0 = (\rho/2)\Omega^2(R^2-0^2)$	p85 eq.3.26a
$(\partial\theta/\partial r) \sim \theta_0(1-0.364)/300$ rad/m	$(\theta_0 - \theta_R)/R$ radians/m average twist per unit length over entire blade. $(\partial\theta/\partial r) \sim 0.636/300 = 2.120 \times 10^{-3}$/m. <Using fig.3.19 and graphically finding the slope at the root of ~-1.1, $(\partial\theta/\partial r) \sim 1.1/300 = 3.7 \times 10^{-3}$/m. Then my torques would be ~ 0.9 of book's torques.>

Then Eq.3.36 is

$$M_0 = I\sigma_0(\partial\theta/\partial r)|_{r=0}$$
$$= I(\rho/2)\Omega^2(R^2)(\partial\theta/\partial r)|_{r=0}$$
$$= (6.333 \times 10^{-7}\ m^3)(\rho/2\ kg/m^3)\Omega^2(300m)^2\ (\theta_0 2.120 \times 10^{-3}/m)$$

Eq.3.37 $= (1.208\ m^5)(\rho/2\ kg/m^3)(\Omega/s)^2\ \theta_0$ hwich does have units $(m)(kg\ m/s^2)$, torque.

In the equation, the inputs are Ω in radians/sec, and θ_0 in radians. In the table the inputs in rpm must be multiplied by 0.10472 to get radians/second, and inputs in degrees must be multiplied by .017453 to get radians.

p88.43 With a change in blade root twist, how long does it take for twist to propagate to the end, and is there overshoot? Damping?
Hwat is reference direction for angles in Fig. 3.20?

p89.17 Fig.3.20 Sol line is coming out of page. Per p88.38, Sol is on axis so torque in fig.3.20b is out of page. Fig.3.20a is pure cyclic. Only half the sail area contributes to lateral force. Is anything gained by rotating other two blades so net force is along diagonal? For fig.3.20b see eq. 3.40. This tilts axis off sun line, or back onto it.

p89.36 Suppose center of mass is not on axis of heliogyro?
Stack several shorter heliogyros for shorter blades but same area. But then they could bump each other during precessing. With sufficient separation, no bumps, but one set could shade another, thereby changing the lightness number β. Another way to transition between spiral and circle.
With heliogyro, can the sail axis always be pointed toward sol? Yes, if blades not at ideal setting only during short maneuvers.

p90.09 “disc solar sail uses spin ...”. So did square IKAROS.
p90.14 Is hoop needed if payload can be attached to center without stays? Weird during changes of α. Combine heliogyro with payload on arm.

p90.16 “payload . . . at centre of disc”, better “payload on axis of disc”.
Payload is attached to the hoop by shrouds, point compression loads.

p90.28 I think you get compression along circumference of hoop by integrating T_0 along a half circle, same way you get hoop tension in a barrel.
From hwere do eqs.3.41 and 3.42 come? Google “vibrating flat things”. Tension in N/m, not /m^2. In eq.3.42, tension at r = 0 is infinite, nuts to that. If spin rate is huge? Even with hoop, spin to reduce billowing.

p91.01 Fig. 3.22 should be here and not on next page.

p91.17 Hwat happens if radius of hoop is smaller than R of sail radius? There should be less billowing within, but hwat about sail outside of ring? From heliogyro seems billowing OK for moderate sail spin. Could have several rings, and shroud to sail center as well.

p91.26 Instead of counter-rotating disc, use disc sail inside ring sail.
Orbital rates change with time, so precession must also.

p92.41 Replace collimating mirror with flat directing mirror. Reflected light is now spread into cone, less efficient but less mass and control problems. Center of mass / center of effort problem. Temperature of secondary mirrors gets high.

p93.17 Fig 3.23 As drawn, the director mirror angle is inconsistent with reflection angle. Note reversed arrowhead to left of “Director”. **n** bisects f_i and f_r. Should **n** be replaced with **m**? No, perfect reflectors. Is **n** perpendicular to the directing mirror? Need inset with directing mirror, f_i, α, **n**, α, f_r.

p93.18 Text assumes sol line is same as sail axis. Does this affect argument? Torques? Does center of force change with mirror angle? Required size of reflecting mirror changes with maximum deflection angle.

p95.14 Consider rectangular collector with parabolic cross section, trough collimator and director.

p94.18 Fig.3.24. Hwat is typical operational α? Secondary mirror α = 45° maximizes tangential thrust.

p95.17. Add “β” to “lightness number β of order unity.”

p96.17 Put dots for HPSS and μ-SS on graph.

p96.19 Table 3.5. Λ_1 Λ_2 Λ_3 are sensitivity constants see p59.40.
Put symbols A m m_{sail} $m_{structure}$ m_p a_0 σ σ_S and η = .85 onto Tab.3.5.

p96.44 “perforations can reduce the sail mass by an order of magnitude”. Without changing sail thickness, sail becomes a net, one that is mostly holes. See fig. 3.5 p67.

p97.07 Add “β” to “sail lightness numbers β of order 10”.

p100.30 To Tab.3.6 add rows for β and η.

p102.37 “Marmon clamp” properly “Marman clamp” from MARx MANufacturing, a band clamp often with explosive release. The Marx Brothers.