Study notes for Chapter Four of *Solar Sailing: Technology, Dynamics, and Mission Applications* by Colin R. McInnes last edit 2016 March 25 MBM

pXXX.YY means page XXX, line YY. Making a guide is useful.

p112.29 "crank" means to change orbit inclination. p21.14

p113.03+ Logarithmic spiral trajectories can be made useful by patching them to planetary circles using intermediate ellipses. See addendum.

p113m Hamilton-Jacobi method

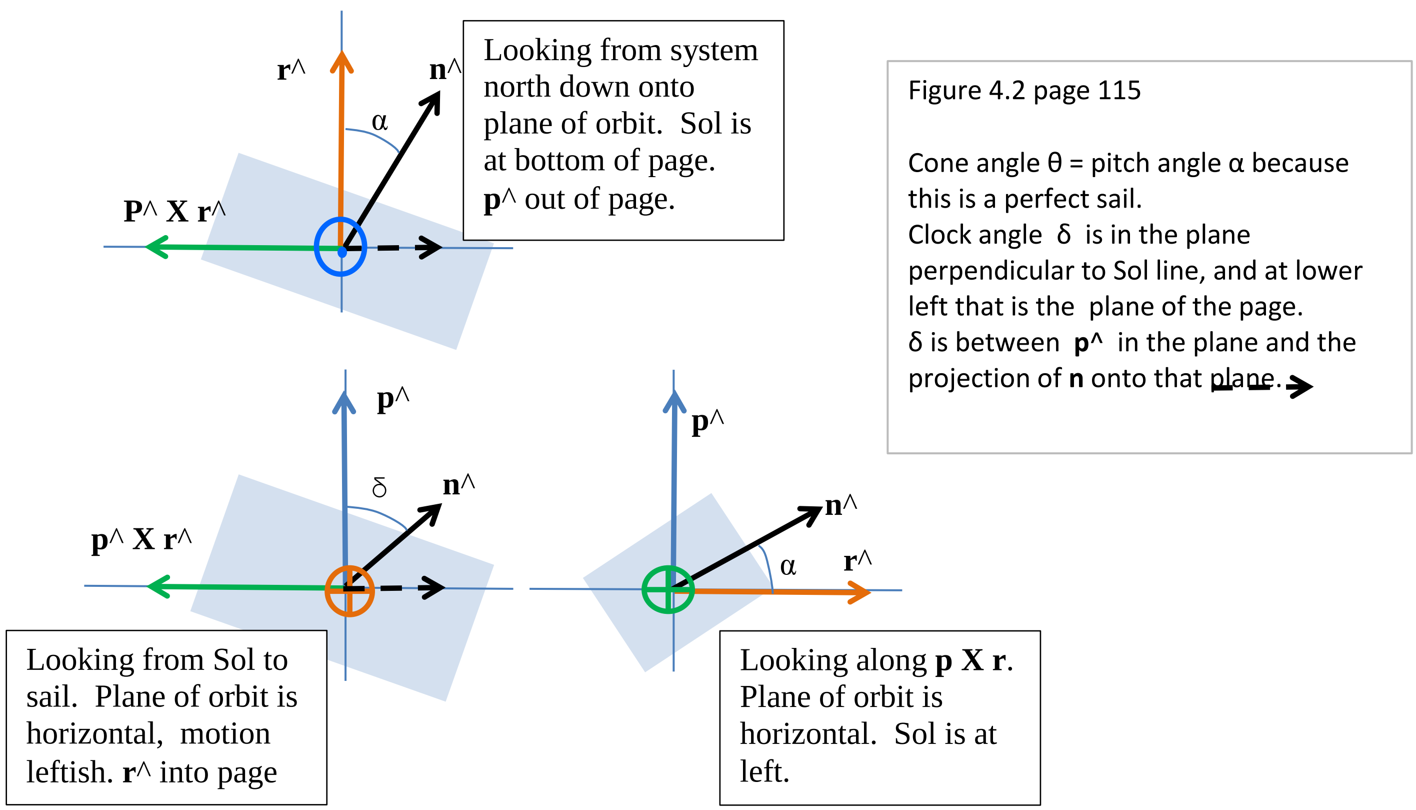
p113.33 Eq. 4.1 should be $\mathbf{R} = (M\mathbf{r}_1 + m\mathbf{r}_2)/(M + m)$. Compute d^2R/dt^2 and use below.

p113.43 Below "(4.2b)" should read "adding" not "subtracting".

p114.22 Think more about inertial reference frames. Does not momentum of reflected light balance momentum change of sail?

p114.42 Approximation $\mu \approx GM$ is used on the RHS of eq. 4.6 and afterward.

p115.06 Some confusion is introduce in the text. The cone angle θ is between Sol line **r**^ and the net force **m**. The pitch angle α is between **r**^ and **n**. Since the discussion is for a perfect sail, **m** = **n** and θ = α. The equations use α but text uses cone instead of pitch.



p115.23 "maximize the component" still leaves residual forces in other directions.

p115.44 Fig.4.2 assumes perfect reflector. Clock angle δ is rotation of **n** about **r** measured from reference **p^** perpendicular to orbit plane. δ is in **p**^ and **p**^ **X r^** plane, fig. 4.2 should make this clear.

Because this is a perfect reflector the cone angle θ = pitch angle = α, measured between **n** and **r**.

p116.03 Eq. 4.9 is eq. 2.20 with u_i = r^. Dot this with q^.

p116.07 In eq.4.10 set δ = δ~ <note tilda> so cos(δ-δ~)=1. Then dot product of *f* with *q* gives force along *q*: f_q = eq.4.10′ = *f*[cos(α)cos(α~)+sin(α)sin(α~)]. To maximize the force against **q** use the negative root. The required cone angle is now 90-(angle with positive root). See p138.

p116.18 Can also write eq.4.12′ tanα* = [-3cos(α~) + √[9$\cos^2$(α~)+8$\sin^2$(α~)]/4sin(α~).

p116.44 In fig.4.3, label vertical axis with α* and horizontal with (α~). Using new eq.4.12 extend graph to left to -90° for negative required cone (pitch) angle (α~).

p117.16 In fig.4.4 note that **q** can point below horizontal. **f** assumes a perfect sail. Add notation for f_q, with the arrow point at the perpendicular along **q.**

p117.38 In fig.4.5, label horizontal axis with α and run from -90° to +90°, vertical axis with f/2PA. Put in numeric values for transverse peak at (35.26° = atan(1/√2).

Radial force is always positive, and symmetric around α = 0 (and around α = 90 if sail is reflective both sides). Transverse is negative (to the right) for α < 0. Total force is root of squares, and so always positive.

p117.40 From. Fig.4.3. Add α and α~ to "sail cone angle α ~~is limited to~~ has a maximum of approximately 35° as the required cone angle α~ reaches 90°." After 90° the back side of sail is turning toward Sol.

p118.14 In fig.4.6, ♈ (Aries) designates a reference direction, hwere θ is zero. Angle θ is in plane of ecliptic, measured from ♈. ϕ is vertical angle measured from ecliptic to sailcraft position vector **r**. At the sailcraft, **r**^, **θ**^, and **ϕ**^ seem to correspond to **r**^, **p**^ X **r**^, and **p** from fig.4.2.

p118.17 Add α~ and α* to "As the required cone angle α~ increases there is a sail cone angle α* which ..."

Again, sometimes it is not the transverse force hwich must be maximized, so α~ may be other than 90°.

"From eq. (4.12) it ...α~ = 90° the", since tan90° = ∞ it is easier to use the alternate eq. 4.12′ to see that the limit as (α~) -> 90° is 1/√2 ≈ 35.264. Radial force goes as $\cos^3$, transverse as $\cos^2$sin per eq. 4.10.

p118.35 "using eq.4.7 it is found that ..." Arrange eq. 4.6 to

Eq. 4.6′ $d^2\mathbf{r}/dt^2 = (-\mu/r^2)\mathbf{r}\hat{} + (\beta\mu/r^2)(\mathbf{r}\hat{}\bullet\mathbf{n})^2\mathbf{n}$.

Note (**r**^•**n**) = cosα. Put this and **n** from eq.4.7 into eq 4.6′:

$$d^2\mathbf{r}/dt^2 = (-\mu/r^2)\mathbf{r}\hat{} + (\beta\mu/r^2)(\cos\alpha)^2[(\cos\alpha)\mathbf{r}\hat{} + (\sin\alpha)(\cos\delta)\mathbf{p}\hat{} + (\sin\alpha)(\sin\delta)(\mathbf{p}\hat{}\ \mathbf{X}\ \mathbf{r}\hat{})]$$

Consolidate, noting **p**^ is **ϕ**^ and (**p**^ **X** **r**^) is **θ**^

$$d^2\mathbf{r}/dt^2 = [(-\mu/r^2) + (\beta\mu/r^2)(\cos\alpha)^3]\mathbf{r}\hat{} + [(\beta\mu/r^2)(\cos\alpha)^2(\sin\alpha)(\sin\delta)]\boldsymbol{\theta}\hat{} +$$

$$+[(\beta\mu/r^2)(\cos\alpha)^2(\sin\alpha)(\cos\delta)]\boldsymbol{\phi}\hat{}.$$

These are the RHS of eqs. 4.13abc.

The LHS of eqs. 4.13abc is $d^2\mathbf{r}/dt^2$, with some confusion over ϕ measured from the polar axis or from the central plane. The confusion causes an interchange of (sinϕ) and (cosϕ), sometimes with a change of sign from taking a derivative. See Wolfram Mathworld. Also note ∂**r^**/∂r =0, ∂**r**^/∂θ = (sinϕ)**θ**^ et cetera.

eq. 4.13a, from eq. 4.6 equate coefficients of **r**^,
eq. 4.13b, from eq. 4.6 equate coefficients of **θ**^,
eq. 4.13a, from eq. 4.6 equate coefficients of **ϕ**^.

p118.37 On RHS of eq.4.13a move exponent. ($\cos^3\alpha$).
p118.40 On RHS of eq.4.13b move exponent. ($\cos^2\alpha$).
p118.42 On RHS of eq.4.13c move exponent. ($\cos^2\alpha$).

p119.15 In fig.4.7,
♈ (Aries) is a reference direction in the plane of the ecliptic.
i is tilt of sailcraft orbit relative to ecliptic plane.
N and N′ are ascending and descending nodes hwere orbit crosses elliptic plane.
A is aphelion distance.
P is perihelion distance. A and P are on the orbital ellipse and collinear with Sol.
Ω is longitude of A, measured along ecliptic from ♈ to N.
ω is argument of perihelion measured from N to P.
r is vector from Sol to sailcraft.
f is true anomaly, angle along orbit from P to sailcraft's position **r**.
S, T, and W are radial, transverse, and crank force components on sailcraft
From these we get
p119.43 $a = (r_A + r_P)/2$.
p119.44 $e = (r_A - r_P)/(r_A + r_P)$.
p120.19 $p = a(1 - e^2)$ semi-latus rectum.
$\mu = G(M+m)$, gravitational parameter.
$n = \sqrt{(\mu/a^3)}$ mean motion or period
τ = time of perihelion passage
t = time sailcraft is at **r**.
$M = n(t-\tau)$ mean anomaly
See diagram in ELLIPSE EQUATIONS.

p120.23 On RHS of eq.4.15a move exponent. ($\cos^3\alpha$).
p120.26 On RHS of eq.4.15b move exponent. ($\cos^2\alpha$).
p120.27 On RHS of eq.4.15c move exponent. ($\cos^2\alpha$).
S, *T*, and *W* are forces, implicitly per unit mass, as are many values.

p120.35 "equinoctial" means WRT celestial equator co-ordinates.

p121.20 "nett" is British for "net".

p121.30 eq.4.16 is eq.4.6 with **n**=**r^**.

p122.02 In eq.4.19 T is now orbital period, and not parameter from p120.25.

p122.15 In eq.4.20b, *E* is energy per unit mass.

p122.20ff The things that look like karp symbols are just < and >

p123.21 Label parabolic orbit ($\beta = 1/2$) and hyperbolic orbits ($\beta > 1/2$).

p123.24 "positive effective solar gravity" means repulsive force.

p123.30 Add "eq. 4.17 and" to "Therefore using eq.(4.17) and eq.(4.19) it".

p121.33 eq.4.23 comes from eq.4.19 with a = r = constant since the orbit is circular, and using $u\sim = u(1-\beta)$.

p123.35 "decoupled" means period also depends on β instead of just on r.

p124.21 In fig.5.9, the hatched lines mean there are no curves below the curve labeled $\beta = 0$. The lower dashed line shows the 25 day rotation of Sol.

p124.44 Altho the text refers to R as being one au, in the rest of sec. 4.3.2.3. and sect 4.3.2.3 R is radius of any initial circular orbit.

p125.17 Change ΔV to Δv_S to be consistent with eq.4.34. The sail is jettisoned at aphelion r before the impulse resulting in addition Δv_S. r becomes new circular distance.

p125.37 eq.4.28a semi-major axis $a = (R + r)/2$.

p125.39 eq.4.28b r is aphelion distance for outer planets, or perihelion distance if you leave circular orbit radius R and apply v_S to slow sailcraft. Jettison at r, and that is now the new circular distance.

p125.43 eq.4.29 $e = 1 - ((1-2)/(1-\beta) = \beta/(1-\beta)$.

Label "sail jettison" and "apogee impulse". Note that semi-major axis of connecting Hohmann orbit is $a = (r_1 + r_2)/2$.

p125m Per normal Newtonian mechanics, the speed^2 of the furled sail in circular orbit at radius r_1 is $\mu\, r_1$. This is also the speed^2 of the deployed sail in elliptical orbit with perihelion r_1 and effective gravitational parameter $\mu\sim$. In eq. 4.20a substitute r_1 for r on the RHS, and equate the two speeds. This is eq. 4.26.
a is the semi-major axis of the new orbit. To get eq. 4.27, solve for a and use eq. 4.17 $\mu\sim = \mu(1-\beta)$ to replace $\mu\sim$. Cancel μ.

p125b Eq. 4/29 reduces to $e = \beta/(1-\beta)$, hwich should be added to the text.

p126.2 Change "Then using eq.(4.28b), the aphelion ..." to "Then dividing eq.(4.28b) by eq. (4.28a) and re-arranging, the aphelion ..."

p126.29 Change paragraph after eq. 4.33 from "Finally, the Δv required ..." to "Finally, the delta vee required ..."

p126.33 Sail speed at aphelion

$$v_A^2 = [\mu(1-\beta)][(2/r) - 1/((R+r)/2))] =$$
$$= [\mu(1-\beta)][(2/r) - (2/(R+r)))] = \qquad \text{Using eq.4.32 } \beta = (r-R)/(2r)$$
$$= 2[\mu(1-\beta)]\frac{[R+r\ -r]}{[r(R+r)]} = 2[\mu(1-\frac{(r-R)}{(2r)})]\frac{[R+r\ -r]}{[r(R+r)]} = 2[\mu\frac{(2r-(r-R))}{(2r)}]\frac{[R+r\ -r]}{[r(R+r)]}$$
$$= [\mu\frac{(r+R))}{(r)}]\frac{[R]}{[r(R+r)]} = \frac{[\mu][R]}{(r)[r]} \quad \text{so } v_S = \sqrt{([\mu/r][R/r]}\text{ . Since at } r,\ v_{circ}\ \sqrt{[\mu/r]}$$

$$\Delta v_S = v_{circ} - v_A = \sqrt{[\mu/r]} - \sqrt{[\mu/r][R/r]} = \sqrt{[\mu/r]}\{1 - \sqrt{[R/r]} \quad \text{finally, eq.4.34}$$

$$[(1/r) - (1/(R+r)))].\ \ )$$
$$= 2[\mu(2/2\ -(r-R)/(2r)][(1/r) - (1/(R+r)))]$$
$$= [\mu(2\ \ -(r-R)/(\ r)][(1/r) - (1/(R+r)))]$$
$$= [\mu(2r/r-\ (r-R)/(r)][(1/r) - (1/(R+r)))]$$

Moving outward, sail and structure are dropped before applying impulse Δv_S. If moving inward, the impulse is applied at aphelion of the transfer ellipse, and must accelerate sail and structure as well as payload. Jettison sail at perihelion.

Differentiate 4.34 wrt R, set equal to zero, with result $r = 4R$, so the solar sail has a maximum Δv_S at 4 au as shown in fig. 4.11a. Hohmann orbit has max at 15.58R, best found numerically.

p127.22 In fig.4.11, R = 1 au, Terra orbit. Label Hohmann maximum at 15.58 au and sail maximum at 4R. "Final Orbit" in r au. Add $r_{Mars} = 1.52$ au, $r_{Ceres} = 2.76$ au, $r_{Jupiter} = 5.20$ au, $r_{Saturn} = 9.54$. $r_{Uranus} = 19.18$ au.

p127.34 Still need engineering comparisons between two impulse Hohmann orbit and sail with single impulse. Trade payload mass against launch weight.

p127.42 "nett" is British, "net" American.

p127.43 Replace "remove" with "offset".

p128.18 In practice sailcraft drops toward Sol, and sets sail at perihelion speed rather than first going to circular orbit.

p129.20 In fig.4.12, label parabolic and linear orbits.

p130.04 "Hyperbolic excess" refers to the speed a body on escape trajectory would have at infinity. For a parabola, hyperbolic excess is zero. For a hyperbola it will be greater than zero.

Note bene: it is possible to go from circular orbit to logarithmic spiral or vice versa using just the sail, with no impulse required. Orbits are patched, not matched. See notes in appendix.

p130.12 "clock angle δ of 90°" is from fig. 4.2 p115 not fig.4.6. (Compare with $\phi = 0$ fig. 4.6 p118) It means staying in the same plane, and cone angle α (sun direction to net force, p130, perfect reflector, text p90) is the same as pitch angle (sun direction to net force same as sail normal).

p130.18 eq.4.37a should start $d^2r/dt^2 = \ldots$, not d^2r/dr^2.

p130.20 In eq. 4.37b, move exponent to $\cos^2\alpha$. It comes from eq. 4.13b with $\delta = 90°$ on RHS and $\phi = 0$ on LHS.

p130.25 Insert "a" in "pitch angle α, a particular"

p130.40 In eq.4.40a, on RHS, exponent of cosine should be three: $\cos^3\alpha$.

To derive eq.4.40a, start with eq. 4.47b,

$r(d^2\theta/dt^2) + 2(dr/dt)(d\theta/dt) = (\beta\mu/r^2)(\cos\alpha)^2(\sin\alpha)$ substitute with eq. 4.39a

$r(d^2\theta/dt^2) + 2(r(\tan\gamma)(d\theta/dt)(d\theta/dt) = (\beta\mu/r^2)(\cos\alpha)^2(\sin\alpha)$ 4.40b′

$r[(d^2\theta/dt^2) + 2(\tan\gamma)(d\theta/dt)^2] = (\beta\mu/r^2)(\cos\alpha)^2(\sin\alpha)$ eq.4.40b.

p131.14 In fig. 4.13, McInnes labels the angle **γ** as the flight angle between **θ**^ and **v**, hwile angle between **v** and **r** is zenith angle. Others use **γ** as zenith angle and **ϕ** as flight angle.

p131.20 Combine corrected eqs.4.40. Multiply 4.40b′ by (tanγ), then subtract4.40a′

4.40b $r(d^2\theta/dt^2)(\tan\gamma) + 2r(d\theta/dt)^2)(\tan\gamma)^2 = (\beta\mu/r^2)(\cos^2\alpha)(\sin\alpha)(\tan\gamma)$

-4.40a $\underline{r(d^2\theta/dt^2)(\tan\gamma) + r(d\theta/dt)^2)(\tan\gamma)^2 - r(d\theta/dt) = (\beta\mu/r^2)(\cos^3\alpha) - (\mu/r^2)}$

$r(d\theta/dt^2)(\tan^2\gamma) + r(d\theta/dt)^2 = (\beta\mu/r^2)(\cos^2\alpha)[(\sin\alpha)(\tan\gamma)-(\cos\alpha)] + (\mu/r^2)$

$r(d\theta/dt^2)(\tan^2\gamma) + r(d\theta/dt)^2 = (\mu/r^2)\{\beta(\cos^2\alpha)[(\sin\alpha)(\tan\gamma)-(\cos\alpha)] + 1\}$

$r(d\theta/dt^2)(\tan^2\gamma) + r(d\theta/dt)^2 = (\mu/r^2)\{1 - \beta(\cos^2\alpha))[(\cos\alpha) - (\sin\alpha)(\tan\gamma)]\}$

$r(d\theta/dt^2)[(\tan^2\gamma) + 1] = (\mu/r^2)\{1 - \beta(\cos^2\alpha))[(\cos\alpha) - (\sin\alpha)(\tan\gamma)]\}$

$r^3(d\theta/dt^2)[1/(\cos^2\gamma)] = (\mu)\{1 - \beta(\cos^2\alpha))[(\cos\alpha) - (\sin\alpha)(\tan\gamma)]\}$

4.41 $r^3(d\theta/dt^2) = (\mu)\{1 - \beta(\cos^2\alpha))[(\cos\alpha) - (\sin\alpha)(\tan\gamma)]\}(\cos^2\gamma)$

Over an entire orbit, eq.4.41 is $r^3/(2\pi/T)^2$ = constant, indeed Kepler-like.

p131.25 Removing the dot notation, $v_\theta = r(d\theta/dt)$. Then

p131.28 Eq4.42 $v_\theta = r(d\theta/dt) = \sqrt{(\text{eq.4.41}/r)}$.

p131.33 Removing the dot notation, and using eq.4.39a, and then eq.4.42 so that $v_r = (dr/dt) = (\tan\gamma)r(d\theta/dt) = (\tan\gamma)v_\theta$ = eq.4.43.

p131.40 To get eq.4.44, add squares of eqs.4.42 and 4.43, take root.

p131.42 Still need to show $v(r) < v_{circular}$.

p132.20 Fig.4.14 add α to horizontal axis and γ to vertical axis. Add line for β = 0.99. If sin γ = √(2/3)the lines "cross" at about (26°, 55°) with β ≈ 0.58. For β < 0.58 there exist γ such that there do not exist α for hwich eq. 4.45 is true. For β > 0.58 there exist α such that there do not exist γ for hwich eq. 4.45 is true.

p132.25 Change to "~~between~~ among" α,β,γ". γ is flexible vis a vis α.

p132.28 Hwence comes eq.4.45? I need help. It has max on LHS hwen $\cos\gamma = 1/\sqrt{3}$, or $\gamma = 54.74°$. Then max LHS = $\sqrt{2}/4 = 0.35355$. With this LHS β has a maximum at $\cos 2\alpha = (24 + \sqrt{657})/81$, or $\alpha = 26.106$.

Multiplying the LHS of eq.4.45 by $(1/\cos^2\gamma)/(1/\cos^2\gamma)$ and using the trig identity $(\tan^2\gamma) + 1 = (1/\cos^2\gamma)$ makes the transformation to
eq.4.45' $(\tan\gamma)/[2+(\tan^2\gamma)] = \beta(\cos^2\alpha)(\sin\alpha)/[1- \beta(\cos^3\alpha)]$. This is quadratic in $(\tan\gamma)$, and the solution can be substituted into eqs.4.41 and .44 so that they are functions only of α and β. Use negative tangent for inbound, positive tangent for outbound.

p132.42 Put $(1-\beta\cos^3\alpha)$ from eq.4.45 into eq.4.43 after distributing the $\beta\cos^2\alpha$, use $\Delta v_r = dr/dt$, and separate variables to get eq.4.46.

p133.03 Derive eq.4.47. α and γ are constant. $1/\tan\gamma = \cot\gamma$, et cetera. It works.

p133.09 "Since eq.4.45 is implicit ..." Explicit, see note p132.28.

p133.13 From the above, the approximation of eq.4.48 is not needed.

p133.30 Eq. 4.51 should have $2/\cos^3\alpha^*$ and not $2/\cos^2\alpha$, and that took way to much effort to establish. Thank you, ErinK. Can solve for β in terms of $(\cos\alpha^*)$ by multiplying top and bottom by $\cos^2\alpha$

$$\beta = (2/(\cos^3\alpha^*))[\ (\cos^2\alpha^*) - 2(\sin^2\alpha^*)\ ]/[2(\cos^2\alpha^*) - (\sin^2\alpha^*)\ ] =$$
$$= (2/(\cos^3\alpha^*))[\ (\cos^2\alpha^*) - 2(1-(\cos^2\alpha^*))]/[2(\cos^2\alpha^*) - (1 - \cos^2\alpha^*)] =$$
$$= (2/(\cos^3\alpha^*))[3(\cos^2\alpha^*) - 2\ ]/[3(\cos^2\alpha^*) - 1\ ] =$$
$$= [6(\cos^2\alpha^*) - 4\ ]/[3(\cos^5\alpha^*) - (\cos^3\alpha^*)\ ] =$$

p133.31 "may be solved numerically ..." As long as a numerical solution is needed, substitute for $\tan\gamma$ in eq.4.47 and solve that numerically.

p133.39 "significant increase" if transferring between circular and logarithmic orbits with an impulse and not patching intermediate ellipses.

p134.20 Add *α, β,* and *γ* to fig.4.15.

p134.23 Tab.4.2. Lots of misprints. Note that $\alpha_0 = \beta GM/r_T^2 = 5.931\beta$, a_0 in mm/s

Table 4.2

B	Hohmann	β = 0.05	β = 0.10	β = 0.125	β = 0.15	β = 0.20
a_0 mm/s^2	-	0.297	0.593	0.741	0.890	1.186
α deg	-	34.98	34.68	34.52	34.36	34.01
γ deg	-	2.27	4.67	5.93	7.24	9.99
T days	259.	873.8	430.1	341.2.	281.8	207.3
Δv_1 km/s	2.95	1.23	2.44	3.09	3.76	5.17
Δv_2 km/s	2.65	1.00	1.98	2.50	3.04	4.19
Δv_T km/s	5.60	2.241	4.42	5.59	6.80	9.36

For β>~0.125, $\Delta v_T > \Delta v_H$. Huh? Less efficiency from extra push? Part of the impulse goes to cancel the orbital velocity, hugely inefficient, and part goes to change direction, even worse. In section4.3.2.3 β=0.1711 reaches Mars with Δv2 = 7 km/s, still bad. Alternatively, set α = zero, then α =90°, and then to α of the spiral. It takes good timing to patch orbits, but takes no (rocket) impulse. Travel times are much longer. See appendix.

A Hohmann orbit is optimized for rockets with minimum impulse, hwilst spiral is optimized for simple steering law, flight angle γ. Apples and oranges.

p134.42 "insensitive to small variations in sail pitch angle". Add "around optimum of 35.xx°".

p135.20 To fig.4.16, add α and T to axes. Add lines for $\beta>0.15$. $\beta=0.05$ is off top of graph.

p135.44 Add monthly tic marks to fig.4.17. Add $\beta = 0.05$ to caption. See pp16,17

p136.19 Again, sail can be used to enter spiral. Either with patching ellipses, or the optimized steering laws discussed later.

p137.04 Eq.4.53 f is true anomaly, $\boldsymbol{f}$ is vector function of S,T,W. S,T,W are the force coefficients per unit mass: S out from sun, T traverse to sun in orbital plane, and W perpendicular to orbit. $W = 0$ for in-plane maneuvers. Z is an orbital element from p120: a e i Ω ω t. The λs are the coefficients of S,T,W from p120t eq.4.14. For example, from eq.4.14b, λ(e) is $[(r^2/\mu)\sin f, (r^2/\mu)(1+r/p)+(r^2/\mu)(re/p), 0]$.

p137.19 Add α and e to line beginning "For changes to ...".
If remaining in the orbital plane, $\delta\sim = \pi/2$.

p137.44 Add perpendiculars from $\boldsymbol{\lambda}$ to axes in fig. 4.18.

p138.03 Compare eq.4.56a with eq.4.12. Here and in spiral orbit calculations the tanα~ version works best, as one wants to go from augmenting the velocity to diminishing it as the flight angle γ goes from positive to negative. (α~ goes from <90 to >90. Eq.4.56a, note lack of +/- before radical

p138.08 "osculating orbital elements", Same position and direction.

p138.28 Misprint, leave off "tan" in eq.4.58a. α~ = π/2 and = 0,π because desired direction is perpendicular to orbital plane.

p138b Eq.4.59a is from eq.4.57. $\boldsymbol{f}$ is in plane of orbit since $\delta\sim = \pi/2$.

p139.7 "common scaling factors S, T, and W" of the coefficients, here $(2r^2)/(\mu(1-e^2)^2)$ from eq.4.14a. $\boldsymbol{\lambda}$ is parallel to $\mathbf{v}$ at least in this case hwere we are changing a, so α* + γ = 90°, not as shown in fig. 4.18.

p139.17 How does two body problem imply eq.4.64? Many following equations use the substitutions $p/r = 1+e\cos f$ and $p = a(1-e^2)$.

p139.36 add β to "lightness number β of 0.05". Compare this to β used to develop force model in earlier chapter. Orbit number is $f/2\pi$, not inverse of that.

p140ff Emphasize that in Figs.4.19-21 graphs start at 1 au, e = 0.2. In Figs.4.19-24 β = 0.05

p140.33 Fig.4.19c has flat top as require α≤35° for operational reasons.
Fig.4.19d. Initial orbit is offset circle, not ellipse, sloppy.

p141.09 Argument of perihelion ω p119t. Are ω and Ω changing so that f's reference direction is changing? This could be awkward.

p141.26 optimum sail cone angle is again α* of eq. 4.56a.

p142.33 In fig.4.20, part of the time eccentricity is decreasing. Hwat if sail goes edge on during this part of the orbit?

p144.10 Eq.4.70b works if $-\pi/2 < (f+\omega) < \pi/2$
Eq.4.70b and eq.4.71 δ* runs from 0 to π, not from −π to +π. See fig.xxxxx

p144.29 From p120 eq.4.14c, max change in I happens hwen f+ω=0 or π (cos = +/- 1). But physically, hwy should force affecting inclination at cosf=0 be zero? Eq.4.71 comes from putting eq.4.15c into eq.4.14c (p120) with δ^* a function of f as in eq.4.70b, and e=0 means r=p. Integrating over 2π (eq.4.72), ω is an irrelevant phase shift, but I think it should be left in eq.4.71.

p144.33 to get eq.4.72, cosδ* is from eq.4.70b. Is not brightness number β important for distant orbits?

If tan α* = 1/√2, sin α* = 1/√3, cos α* = √2/√3. Putting these values into eq.4.73 gives Δi = 4β 2/3 1/√3 = 1.54β ≈ 88β°/orbit.

p144.44 Add "diminishing" to "diminishes with diminishing orbit radius"

P145.35 Fig.4.22d is looking at fig.4.22c from along x-axis (back rite face).

p147.35 Fig.4.24 covers 14 orbits, fig.4.22 only seven. Prefer seven.

p147.40 Eq.4.78 is eq.4.77 with substitutions and multiplied by 180/π.

P148.28 Eq.4.79b is from eq.4.6. Dot notation for /dt is inconsistent with earlier usage, and confusing.

p148.34-44 co-states? I am lost. Hamiltonian.

p148.40 eq.4.80. Should subscript in second term on RHS be $\mathbf{p}_r$ and not $\mathbf{p}_v$?

p149.04-06 Hwence eqs.4.81ab? Replace dot notation with d/dt.

p149.10 Eq.4.82 makes sense if $\partial/\partial\mathbf{r}$ ($\mathbf{p_v}$ + $\mathbf{p_r}$ + $\mathbf{n}$ + $\mathbf{r}$ + r) = (0+0+0+1+r^), except $\mathbf{p_r}\bullet\mathbf{r}$ should be $\mathbf{p_v}\bullet\mathbf{r}$, I think. And how do the time derivations fit in? Chain rule?

p149.32 Transversality condition?

p150.25 Fig.4.25. Hwy does return take longer? Should not orbits be symmetric? Perhaps the total of waiting time at Mars and the return trip time that is minimized.

p151.06 Hwy go past Mars orbit? In the simplest case, start in circular orbit R with furled sail. Speed V_0 = √{μ/R}. Now set sail of lightness number β, with pitch angle α = 0. The circular speed is now V_S = = √{μ(1-β)/R} < V_0. The sailcraft is at perihelion and so must move outward. For other α the radial push still outweighs the transverse push. Arriving at Mars is the mirror image of this maneuver.

p151.12 Section 4.4. At the expense of some blank paper, it would be nice to have equations an figures all on the same page spread.

p151.35 Add a_0 to "sail characteristic acceleration a_0". $\mathbf{l}$ is now Solline vector and $\mathbf{r}$ is used for planet-centered vector.

Negligible air drag at heights >900 km above Terra surface.

p152.02 Need more discussion of third body effects.

p152.19 Discussion assumes that initial orbit is in ecliptic. Is there an inclination between zero and π/2 that is more efficient than either of the extremes?

p152.44 Fig.4.26 Sol is to left, so show Terra sunlit on left side. Orbit starts with f = 0 on the far side of Terra from Sol. Add two more sails at Terra face on to Sol at f = 0 and f = π. Add normal $\mathbf{n}$ to each sail.

p153.07 e ≈ 0 so p ≈ a, r ≈ a. Hwat else changes besides a and e? ω?

From here on S,T,W are called acceleration. On p120 they are called forces, with "per unit mass" assumed.

p153.14 κ is a constant that depends on specific conditions.

p153.26 Note the conditions that carry on thru this section. However, the delta vee to go from LEO to GEO is more than the delta vee to go from LEO to escape. In practice the sail would start from GEO only if catching a ride from another craft going to that orbit.
If there is insufficient delta vee to escape, go for the elliptical orbit with minimum safe perigee and maximum apogee. Orient ellipse so that perigee is at π/2, and sailcraft can spend most of the time face-on to Sol. See p158.25.

p154.33 Fig.4.27 refers to Section 4.2.2.1 on previous pages.
Fig.4.27a Top of scale should be 45,000, not 43,000.
Fig.4.27d Sol is on left, lighten half of Terra. f = 0 is (39000km, 0km).

p155.17 In fig.4.28, θ should be f for consistency. Sol at left, Terra sunny, add sail normals. To be consistent with fig.4.28 and the 180° flip, eq.4.92 should reflect

f =	0(π/4)	1(π/4)	2(π/4)	3(π/4)	4(π/4)	5(π/4)	6(π/4)	7(π/4)	8(π/4)
α =	2(π/8)	3(π/8)	4(π/8)						
			-4(π/8)	-3(π/8)	-2(π/8)	-1(π/8)	0(π/8)	1(π/8)	2(π/8)

p155.22 Eq.4.93 Same problems with flip.

p156.42 To go from LHS eq.4.95 to eq.4.96, take time derivative on RHS of eq. 4.96, then multiply rite term by r/r. Since r is parallel dr/dt, rdr/dt = $\boldsymbol{r}$d$\boldsymbol{r}$/dt = $\boldsymbol{r}$d$\boldsymbol{r}$/dt cos0 = $\boldsymbol{r}$•d$\boldsymbol{r}$.

p157.16 Fig.4.30 Sol on left, sunlit Terra, show **l** (sunline), and α between **l** and **n** (sail normal). Angle f should not be bold. Add α to figure. Refers to sect4.4.2. Looks very similar to fig.4.28, but compare tilt of sail at f = 0 and f = π.

p157.21 Eq.4.96 requires **r** perpendicular to **v**, approximately true here.

p157.23 add per unit mass "orbit energy per unit mass E".

p157.39-43 It is tediously true that eq.4.100 solves eq.4.99. Both can be transformed to $\cos\psi(3\cos\alpha\sin\alpha)=\sin\psi(1+3\sin^2\alpha)$, with α^* indicating optimum.

p158.4 Eq.4.101 probably follows from eq.4.100, eq.4.15b, and fig. 4.30.

p158.25 If GTO is properly aligned, it has the advantage of requiring just enuf apogee kick energy for the sail to avoid air drag at perigee. Or, use the energy needed to go from GTO to Clarke orbit to maximize the apogee. If sail is going into Sol at perigee, it spends so little time there that apogee is barely reduced. This is the simplest steering law. Better (less time to escape) is a steering law to increase apogee hwenever possible. See note p153.26.

p160.18 Fig.4.32 Sol to left, Terra needs half-light.

p160.35 Fig.4.33 Sol is above page shining down. Terra should be fully lit.

p161.33 Fig.4.34d Sol is shining down on page. Terra should be fully lit.

p161.38 Eq.4.106 depends on $e\approx 0$ so that $r\approx a$. This ignores the force component moving orbit plane out-Sol from Terra's center of mass.

p162.12 eq.4.107 and eq.4.108 from eq.4.14a and eq.4.103.

p162.40 In tab.4.3, Δa is per orbit.

p163.13 There is no sec.4.4.4.3. Must mean sec.4.4.2.3.

p164.11 Eq.4.111. da/dt = (da/df)(df/dt). $P = 2\pi\sqrt{a^3/\mu}$. f changes by 2π radians in P seconds. $df/dt = 2\pi/P = \sqrt{\mu/a^3}$.

p164.25 Eq.4.114 6371 km = r_T.

p165.21 In fig.4.35 the diagonal lines are labeled with height above Terra’s surface at 6371 km.

p165.37 T & V are energy per unit mass.

p166.20 Eq.4.117 derives messily from eq.4.116.

p166.29-34 $\dot{\xi}=d\xi/dt$ etc. ?

p166.37 $H=T+V$ is consistent with eq.4.119.

p167.05 Eq4.120 etc. I’m lost.

p168.18 Fig.4.37 has Sol to left. Light on Terra disc. Put in parabolic envelopes (plural?) mentioned p168m. f = 0 starts at (42421km, 0).

p169.14. Compare initial equatorial, ecliptic, and polar orbits attainable for the same launch delta-vee and mass. Then look at time to raise to escape.