Study notes for Chapter Five of *Solar Sailing: Technology, Dynamics, and Mission Applications* by Colin R. McInnes   last edit 2016 April 6 MBM

pXXY means page XX. Y = t/m/b for top/middle/bottom third of page

Chapter Five

p171.12 Non-Keplerian means a continuously powered orbit, specifically a circular orbit (p172.12, p172.23) of radius ρ, usually with the central mass not in the plane of the orbit.

p172.12 Displaced elliptical orbits are weird, probably not stable? Maintaining a give offset distance z may be possible by varying the pitch angle, but then the central force is no long simply $1/r^2$.

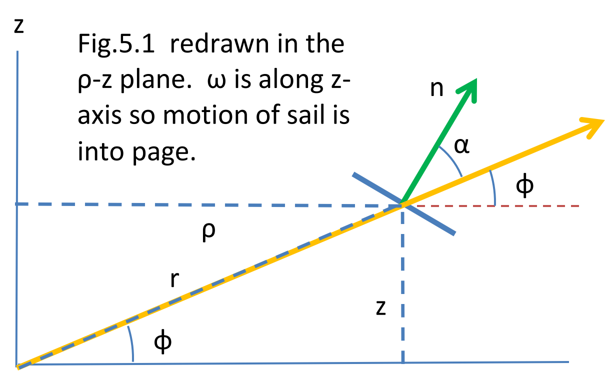
p172.34 how stable is a sail in orbit around expanded Lagrangian point?

p173.04 Add "operation for axially displaced solar sails". Sun-centered non-Keplerian means circular but displaced along orbit axis, Sol no longer in plane of orbit. There are also "forced orbits" with the central mass in the orbital plane but with a non-Keplerian period.

p173.12 Emphasize ρ is constant.

p173.24 Does "Keplerian synchronous orbit" mean synchronous to regular Keplerian orbit? If so "Keplerian-synchronous" may help.

p174.13 Fig.5.1. redrawn. Angle $\tan\phi = z/\rho$. Note that ρ is radius of the displaced orbit.



p174.35 "as defined in section 2.3.3". Eq.5.2a, **a** from eq.2.24 by substituting in μ. *V* is Newtonian physics.

p174.42 Need to confirm eq.5.3b.

p175.05 "Equilibrium" means $dr/dt = 0$, and $d^2r/dt^2 = 0$.

p175.08 Eq.5.5 ∇U **X** **n** = 0 means they are parallel, so ε can be anything.

p175.06 **a** and **n** are parallel so **a** **X** **n** = 0.

p175.24 Eq.5.8 is from eq.5.4 and eq.5.2a.

p175.32,38,40 Confirm eq.5.9, eq.5.10. Could not follow book, so used alternate method.
Using the notation in redrawn fig.5.1, the acceleration due to gravity is
$a^{Grav} = (\mu/r^2)$, and from photon pressure
$a^{Phot} = \beta(\mu/r^2)\cos^2\alpha$.
ω is the constant rotation about the z-axis.
ω~ is rotation speed at radius r if there is no photon acceleration. $(\omega\sim)^2 = \mu/r^3$.
$\tan\phi = z/\rho$ and $\cos\phi = \rho/r$.

Then the accelerations parallel to the z-axis are

$a_z^{Grav} = (\mu/r^2)\sin\phi$

$a_z^{Phot} = \beta(\mu/r^2)\cos^2\alpha\sin(\phi+\alpha)$

For z to stay constant, the accelerations sum to zero.

$$a_z^{Grav} = a_z^{Phot}$$
$$(\mu/r^2)\sin\phi = \beta(\mu/r^2)\cos^2\alpha\sin(\phi+\alpha)$$
$$\sin\phi = \beta\cos^2\alpha \quad \sin(\phi+\alpha)$$
$$\sin\phi = \beta\cos^2\alpha\{\sin\phi\cos\alpha + \cos\phi\sin\alpha\}$$
$$\tan\phi = \beta\cos^2\alpha\{\tan\phi\cos\alpha + \sin\alpha\}$$

Eq.X $\quad \tan\phi = \beta\cos^3\alpha\{\tan\phi \quad + \tan\alpha\}$

The accelerations parallel to ρ are

$a_\rho^{Grav} = (\mu/r^2)\cos\phi$

$a_\rho^{Phot} = \beta(\mu/r^2)\cos^2\alpha \cos(\phi+\alpha)$

For circular motion with radius ρ, the required central acceleration

$$\omega^2\rho = a_\rho^{Grav} - a_\rho^{Phot}$$
$$\omega^2\rho = (\mu/r^2)\cos\phi - \beta(\mu/r^2)\cos^2\alpha \cos(\phi+\alpha)$$
$$\omega^2\rho = (\mu/r^2)\cos\phi - \beta(\mu/r^2)\cos^2\alpha \{\cos\phi\cos\alpha - \sin\phi\sin\alpha\}$$
$$\omega^2\rho/r = (\mu/r^3)\cos\phi - \beta(\mu/r^3)\cos^2\alpha \{\cos\phi\cos\alpha - \sin\phi\sin\alpha\}$$
$$\omega^2\rho/r = ((\omega\sim)^2)\cos\phi - \beta((\omega\sim)^2)\cos^2\alpha \{\cos\phi\cos\alpha - \sin\phi\sin\alpha\}$$

Temporarily writing $W = \omega^2/(\omega\sim)^2$ and remembering $\cos\phi = \rho/r$

$$W\cos\phi = \cos\phi - \beta\cos^2\alpha \{\cos\phi\cos\alpha - \sin\phi\sin\alpha\}$$
$$W = 1 - \beta\cos^2\alpha \{\quad \cos\alpha - \tan\phi\sin\alpha\}$$
$$(W-1) = -\beta\cos^2\alpha \{\cos\alpha - \tan\phi\sin\alpha\}$$

Eq.Y $\quad (1-W) = \beta\cos^3\alpha \{ \quad 1 - \tan\phi\tan\alpha\}$

Dividing Eq.X by Eq.Y

$$\tan\phi/(1-W) = \{\tan\phi + \tan\alpha\}/\{1 - \tan\phi\tan\alpha\}$$
$$\tan\phi\{1 - \tan\phi\tan\alpha\} = \{\tan\phi + \tan\alpha\}(1-W)$$
$$\tan\phi - \tan^2\phi\tan\alpha = \tan\phi(1-W) + \tan\alpha(1-W)$$
$$\tan\phi - \tan\phi(1-W) = \tan\alpha\}(1-W) + \tan^2\phi\tan\alpha$$
$$\tan\phi(W) = \{(1-W) + \tan^2\phi\}\tan\alpha$$

$$\tan\alpha = \frac{\tan\phi(W)}{\tan^2\phi+(1-W)} = \frac{\tan\phi \; (\omega^2/(\omega\sim)^2)}{\tan^2\phi+(1-\omega^2/(\omega\sim)^2)}$$ in agreement with eq.5.10a.

With even more fussiness put eq.5.10a into Eq.X to get eq.5.10b.

p175.44 Making μ unity means it may disappear from equations, just be there implicitly. Check units.

p176.08 Type I: fixed orbital period (fixed ω) for all radii ρ and displacement from central mass *z* (distance from central mass *r*.

p176.44 Figs.5.2, 5.3, and 5.4. The numbers 1-6 next to curves refer to columns in Tab.5.1 and are not themselves β. Sol at center, should be an open circle. Arrows are sail normals. Sail can lengthen period, never shorten it. Eqs.5.10ab contain $(\omega\sim) = (\mu/r^3)$, with $r = \sqrt{\rho^2 + z^2}$. This additional dependence on ρ and z leads to the curves of fig.5.2.

p177.36 eq.5.11 from eq.5.6, using eq.5.9? r^•(eq.5.6)?

p178.20 Numbers in figs.5.2,5.3,and 5.4 come from Tab.5.1. Sol at center. Confusion at z=0 ρ=1. This is Terra's orbit (r=ρ) so β=0 or α=90° should work.

p178.28 Type II: *ω = ω~*, so that the sailcraft at distance r and displacement z maintains distance and orientation relative to a planet.

p178.32 In eq.5.12ab, note that the ration has gone from z/ρ to ρ/z.

p179.01 Type III: For a given z and ρ, choose ω to get the minimum β possible.

p179.44 Numbers in figs.5.2,5.3, and 5.4 come from Tab.5.1, Sol at center.

p180.06 Sec.5.2.2.4 $\omega = \rho^{-3/2}$, units check, see p175.

p180m Sec.5.2.3 Eqs.5.16-5.29. I am lost. I did not put most of these symbols in the list.

p183.37 Section 5.2.3.2. Still lost.

p183.41 "ω = 1 in eq.5.27", ω is part of L in eqs.5.24.

p184.34 To fig.5.6, add to caption. Below C_1 is stable, including C_3. The right-hand side is forbidden. The rest, including C_2, is allowed but not stable. The C_i correspond to extreme Type I orbits, and Types II and III.

p186.20 Fig.5.7. See p185.18

p186.25 In eq.5.34 hwich comes from eq.4.16, add exponent. d^2r/dt^2

p186.32 η is radial perturbation, ξ is transverse. β = 1.
Perturb by $\boldsymbol{r_0} \rightarrow \boldsymbol{r_0} + \boldsymbol{\delta}$, with $\boldsymbol{\delta} = \eta\mathbf{r\hat{}} + \xi\boldsymbol{\theta}\hat{}$.

p186.35 Show eqs.5.35a. By eq.5.34 $d^2\mathbf{r}/dt^2 = d^2r\mathbf{r\hat{}}/dt^2 = -(1-\beta)(\mu/r^2)\mathbf{r\hat{}}$.
Draw $r\mathbf{r\hat{}}$ and $\xi\boldsymbol{\theta}\hat{}$ at 90° to it, so their sum (hypotenuse) is $\surd\{r^2 + \xi^2\}$. Then
$d^2\mathbf{r}/dt^2 = d^2(r\mathbf{r\hat{}} + \xi\boldsymbol{\theta}\hat{})/dt^2 = d^2(r\mathbf{r\hat{}})/dt + d^2(\xi\boldsymbol{\theta}\hat{})/dt^2$ = so that
$d^2(\xi\boldsymbol{\theta}\hat{})/dt2 = d^2(r\mathbf{r\hat{}} + \xi\boldsymbol{\theta}\hat{})/dt^2 - d^2(r\mathbf{r\hat{}})/dt$ =
$-(1-\beta)(\mu/(\surd\{r^2 + \xi^2\})^2) - (-(1-\beta)(\mu/r^2))$ =
$-(1-\beta)(\mu/\{r^2 + \xi^2\}) + (1-\beta)(\mu/r^2)) = -(1-\beta)(\mu)[1/\{r^2 + \xi^2\} - (1/r^2)]$
$= -(1-\beta)(\mu)[r^2 - (r^2+ \xi^2)]/[r^2\{r^2 + \xi^2\}] = -(1-\beta)(\mu)[-\xi^2)]/[r^2\{r^2 + \xi^2\}]$ =
******** try this again, see notes on separate sheet.

Eq.5.35a If you displace the sail a small distance ξ transversely without re-orienting it to the sun-line, the normal now makes an angle of $\alpha = \xi/r$ radians WRT original sun-line. The radial force is then $-(1-\beta\cos^3\alpha)\mu\mathbf{r\hat{}}/r^2$ and the tangential force is $-(\beta\cos2\alpha\sin\alpha)\mu\boldsymbol{\theta}/r^2$. Since α is small $\cos\alpha \approx 1$ and $\sin\alpha \approx \xi/r$. Given β = 1, the tangential force is $-1(1)^2(\xi/r)\mu/r^2 = -\xi\mu/r^3$. This is a restoring force. Note that the radial force is not quite zero any more. The sail will overshoot and oscillate across the original sun-line, but moving ever more sunward. See also eqs.4.37ab with $d\theta/dt = 0$ (just displacement with no residual motion).

Eq.5.35b For β = 1, $d^2\mathbf{r}/dt^2 = 0$ by eq.5.34, and
$d^2(\mathbf{r}+\boldsymbol{\eta})/dt^2 = (d/dt)[d(\mathbf{r}+\boldsymbol{\eta})/dt] = (d/dt)[d\mathbf{r}/dt + d\boldsymbol{\eta})/dt] = d^2\mathbf{r}/dt^2 + d^2\boldsymbol{\eta}/dt^2$
LHS is $-(1-\beta)(\mu/(\surd\{r^2 + \eta^2\})^2) = -(1-\beta)(\mu/\{r^2 + \eta^2\} = 0$ since $\beta = 1$
RHS is $-(1-\beta)(\mu/r^2) + d^2\boldsymbol{\eta}/dt^2 = 0 + d^2\boldsymbol{\eta}/dt^2$ so that $0 = d^2\boldsymbol{\eta}/dt^2$. Maybe

p187.07 F is from p45.07.

p188.11 "active station keeping". Any physical sail will need active control, as its characteristics will never be known exactly, nor will they be stable. This needs more consideration. IKAROS could vary the reflectivity of portions of the sail.

p188.20 In general, sec.5.2.4 loses me.

p188.35 Sail elevation γ defined p174 fig.5.1. Sec.5.2.4.1 lost me.

p189.15 Think of $\delta\gamma$ as "change in gamma".

p190.07 Sect.5.2.4.2 Lost again.

p190.30 "damp asymptotically to non-zero values." Does this imply that solid line of fig.5.9 ends up below the x-axis? In this section, does α vary? I think it must. Then in sec.5.2.4.3 α is indeed fixed.

p190.42 "does not damp errors" may mean orbit is close to desired but not exact.

p191.20 Fig.5.8, same as fig.5.7 (p186) but with control by varying γ. See p193 fig.5.8 as well as fig.5.10.

p191.44 Vertical axis is $\delta\gamma$. The solid line leads from sec.5.2.4.2 to fig.5.8 (α varies), and dashed line goes with sec.5.2.4.3 and fig.5.10 (α fixed).

p192.02 Compare fig.5.1 p.174 and its redrawn version in these notes. The angle between r and ρ is ϕ, and $\gamma = \phi + \alpha$. $\cos\phi = \rho/r$ and $\sin\phi = z/r$. Then add parentheses to eqs.5.50ab (similar to discussion of p175) to get:

p192.07 eq.5.50a $a_\rho = (\beta/r^2)\cos^2\alpha\cos\phi = (\beta/r^2)\cos^2\alpha[\ (\cos\alpha)(\rho/r) - (\sin\alpha)(z/r)\ ]$

p192.10 eq.5.50b $a_z = (\beta/r^2)\cos^2\alpha\sin\phi = (\beta/r^2)\cos^2\alpha[\ (\sin\alpha)(\rho/r) + (\cos\alpha)(z/r)\ ]$, noting the plus sign instead of a minus sign.

p192.12ff Lost. Eq.5.50ab

p193.18 eq.4.56. Restate as $\alpha = \gamma - \mathrm{atan}(z/\rho) = \gamma - \phi$

p193.23 How to get from eq.5.56 to eq.5.57?

p193.29 Compare fig.5.7 and fig.5.10

p193.42 "identical, but have different orientation" is not obvious, unless perhaps restricted to circular orbits with same forcing acceleration.

p193.44 "retrograde" is not necessary. In fig.5.12 (p196), from Orbit I around +z-axis transfer to Orbit II around +y-axis. After 1/4 orbit, transfer to Orbit VI (-x-axis), and after another 1/4 orbit to Orbit III (-z-axis) in same direction as Orbit I. Better if orbit numbers followed same pattern as fig.5.11 p195.

p194.20 Compare fig.5.10 with fig.5.7.

p194.30 "in fig.5.11" See also fig.5.1 p174

p194.32 Change to "The orbit inclination $i = \phi$ is obtained ..."

p194.36 "argument of ~~pericentre~~ periapsis (or in this case perihelion) of 270°". 270° from hwat reference? In any case, perihelion is aphelion plus 180°.

p194.37 "patch always occurs at aphelion" again implies that "non-Keplerian" means "circular but displaced from solar plane".

p194.38 "force exerted on the solar sail is always normal to the solar sail velocity vector" again implies that "non-Keplerian" means "circular but displaced from solar plane".

p194.41 Eq.5.58 has no mass term, so this is energy/unit mass, as mentioned elsewhere. E/m = KE/m + PE/m. It is easier to get the semi-major axis "a" from vis viva equation: $v^2 = 2\mu/r - \mu/a$. As the sail turns edgewise to Sol the only force is from gravity. Put $v = \omega\rho$ and $r = r_A = \sqrt{\rho^2 + z^2}$ into vis viva equation to get $\omega^2\rho^2 = 2\mu/\sqrt{\rho^2 + z^2} - \mu/a$, and rearrange to eq.5.59.

p195.16 Fig.5.11. Orbit II around + x-axis. In fig.5.12 Orbit II is around + y-axis. Text references to z_1 z_2 ρ_1 ρ_2 ω_1 ω_2 are in sec.5.2.5.2. See note p193.44.

p195.19 Eq.5.59, see note p194.41.

p195.24 eq,5,60 $a(1+e) = r_A = \sqrt{\rho^2 + z^2}$ since energy of two orbits is the same.

p195.36 "rectilinear" means sail is on axis and remains on axis.

p195.44 Eq.5.62 from $v_1 = v_2$. There is no impulse as sail goes edge on, and speed stays the same at the patch.

p196.15 Fig.5.12 should have the same orbit nomenclature as fig.5.11. Suggest x- y- z- axes correspond to Orbits I II III, and the negative axes be IV V VI.

p196.22 $\rho_1 = \rho_2$ implies $z_1 = z_2 = \rho_1 = \rho_2$.
Verify that eq.5.63 follows from eq.5.10b.

p196.26 It is not necessary that orbit III is retrograde to orbit I. From orbit I, do a quarter of orbit II, then a quarter of orbit VI on the –x-axis face. Here using the labels of fig.5.12. See notes p193.44.

p196.31 Emphasis, "non-Keplerian" orbit is circular. If sailcraft is face-on to Sol, orbit will be Keplerian with reduced gravitational parameter $\mu(1-\beta)$.

p197.21 As in sec.5.22, I do not follow the derivation in sec.5.3.2, and use an alternative derivation for eqs.5.73ab.

p197.44 Fig.5.13, displaced orbit is perpendicular to z-axis, and z-axis is in anti-Sol direction. z is also displacement distance along that axis, with $z^2 + \rho^2 = r^2$. Not sure of reference direction for θ. ℓ is unit vector along sun-line, parallel to z-axis. For continuity, this figure and fig.5.1 should have axes oriented in the same directions. See redrawn version in notes for p199.

p198.02 Add "a_0" to "characteristic acceleration a_0."

p198.08 eq.5.64 **a** is acceleration from photon force.

p198.12 eq.5.65a κ is a constant, the photon acceleration of the sailcraft with $\alpha = 0$. I think $\kappa = \beta\mu/r^2$.

p199.15-18 Alternative derivation for eqs.5.73ab
Redraw fig.5.13 with y and z in plane of page, +x-axis going into page.

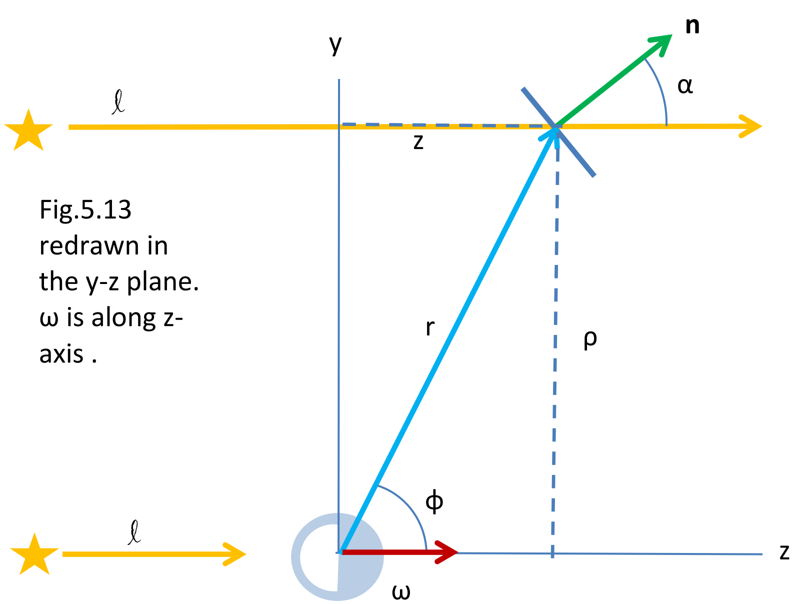


Fig.5.13 redrawn in the y-z plane. ω is along z-axis .

$\sin\phi = \rho/r$ and $\cos\phi = z/r$ $M_{Terra}G = \mu_T$ $(\omega\sim)^2 = \sqrt{\{\mu_T/r^3\}}$

If $\alpha = 0$, κ is the photon acceleration of the sailcraft along ℓ, and $\kappa\cos^2\alpha$ is the acceleration along **n**. Then the components in the y and z directions are $a_y^P = \kappa\cos^2\alpha\sin\alpha$ and $a_z^P = \kappa\cos^3\alpha$

The gravitational acceleration of Terra on the sailcraft is $a^G = \mu_T/r^2$, and the components in the y and z directions are $a_y^G = (\mu_T/r^2)\sin\phi$ and $a_z^G = (\mu_T/r^2)\cos\phi$.

Since the orbit offset z is constant, the accelerations along that axis balance.

(eq.A) $\kappa\cos^3\alpha = a_z^P = a_z^G = (\mu_T/r^2)\cos\phi = (\mu_T/r^3)z$

The sailcraft is in circular orbit around the z-axis with radius ρ and angular speed ω. The $\omega^2\rho$ = centripetal acceleration = $a_y^G - a_y^P = (\mu_T/r^2)\sin\phi - \kappa\cos^2\alpha\sin\alpha$ so that

(eq.B) $\kappa\cos^2\alpha\sin\alpha = (\mu_T/r^2)\sin\phi - \omega^2\rho = (\mu_T/r^3)\rho - \omega^2\rho$

Divide eq.B by eq.A to get

$$\tan\alpha = \frac{[(\mu_T/r^3)\rho - \omega^2\rho\ ]}{[(\mu_T/r^3)z\ ]} = \frac{\rho}{z}\frac{[(\mu_T/r^3) - \omega^2]}{[(\mu_T/r^3)]} = \frac{\rho}{z}[1 - \omega^2/(\omega\sim)^2] \qquad \text{(eq.5.73a)}$$

Since $(1/\cos^2\alpha) = 1 + \tan^2\alpha$, $(1/\cos\alpha) = [1 + \tan^2\alpha]^{1/2}$. Solve (eq.A) for κ and substitute for $(1/\cos\alpha)$ to get (eq.5.73b).

In cylindrical co-ordinates, del ≡ $\boldsymbol{\rho}$**^**$(\partial/\partial\rho)$ + $\boldsymbol{\theta}$**^**$(1/\rho)(\partial/\partial\theta)$ + **z^**$(\partial/\partial z)$. Note also that **$\partial\boldsymbol{\rho}/\partial\boldsymbol{\theta}$ = $\boldsymbol{\theta}$^ and $\partial\boldsymbol{\theta}/\partial$ $\boldsymbol{\theta}$ = -r^.** All other partials = 0. And **$\boldsymbol{\rho}$^**X**z^** = **-$\boldsymbol{\theta}$^**, **$\boldsymbol{\theta}$^**X**z^** = **$\boldsymbol{\rho}$^**, **$\boldsymbol{\rho}$^**X**$\boldsymbol{\theta}$^** = **z^**.

p198.34 "two terms of eq.5.68 must vanish", since in the required equilibrium **r** is constant. Thus del$U$ = **a** = del$V$ + delΦ = del$(-\mu/r)$ + del(**ω**X**ω**X**r**) by eqs.5.65b and5.66b. At any rate, del$U$ is parallel to **a** hwich is parallel to **n** by eq.5.65a. Thus Eq.5.69.

p198.37 Eq.5.69 **a** parallel to **n** by eq.5.65a.

p198.43 Eq.5.70 loses me.

p199.09 Eq.5.72 Use del in cylindrical co-ordinates and definition of U p198m (or above), and mess with the algebra.

p199.25 Hwat are the appropriate "rescaling"? Lost, eqs.5.71-5.73b.

p199.30 add ρ and z to "orbit radius ρ and displacement distance z."

p199.32 $\omega = (\mu^{1/2})r_0^{-3/2}$. Even if units are chosen so that (μ)=1, it is easier to follow the derivations if it is included.

p199.36 Add κ "characteristic acceleration κ ..."

p199.43 "surfaces expand and contract." In fig.5.14 surfaces 3,4 and 5 are always connected at (ρ=±30, z=0). Surfaces 1 and 2 have the double lobe connected as part of the toroidal surface and the separate section on the rite.

p200.20 Fig.5.14 shows surfaces of equal κ, identified in tab.5.2. Sol to left. Planet should be half lighted. Add to caption that, defining period is based on $r_0 = 30r_{planet}$. Also, include the graph just to the left of the planet to show that there are no day-side solutions. At (ρ=±30, z=0) sail is edge-on to Sol. For C_1 see p205m.

p200.26 Tab.5.2. Again, based on $r_0 = 30r_{planet}$. Add κ to caption "acceleration κ for Mercury ..."

p200.33,37,41 Hwy a_0 and not κ? Have not confirmed numbers in table.

p201.06 Add √{μ} "that $\omega = \sqrt{\mu}\rho^{-3/2}$ ", remembering that here √{μ} = 1.

p201.09 "cylindrical" is general, topologically similar.

p201.14 Get eq.5.74a by putting $\omega^2 = \mu\rho^{-3}$, $\tilde{\omega}^2 = \mu/r^3$ into Eq.5.73a, remembering that $\mu = 1$. Then $\tan\alpha = (\rho/z)[1-r^3/(\rho^3)]$. Remembering $r = (\rho^2 + z^2)^{1/2}$, $\tan\alpha = (\rho/z)[1-(\rho^2+ z^2)^{3/2}/(\rho^3)] = (\rho/z)[1-(\rho^2/\rho^2 + z^2/\rho^2)^{3/2}] = (\rho/z)[1-(1 + z^2/\rho^2)^{3/2}]$.
Deriving eq.5.74b is a similar process, not forgetting the lonesome z on the right.

p201.21 $\kappa = 0$ means unforced orbit, strictly Keplerian.

p201.44 Fig.5.15 C_2 defined p206m. Show part of graph Solward of planet.

p202.01 "with $\omega = \sqrt{\mu}\rho^{-3/2}$ "

p202.02 "large displacements" means large z.

p202.08 Add "in eq.5.73b" to "with respect to ω to zero in eq.5.73b". This does imply $\omega=\tilde{\omega}$.

p202.25 From eq.5.76b substitute for r using $r = (\rho^2 + z^2)^{1/2}$ and solve for ρ to get eq.5.77. Then remember from p199 eq.5.73a that $\tilde{\omega}^2 = \mu/r^3$ and $\mu = 1$.

p202.30 In eq.5.77 set $d\rho/dz = 0 = (1/2)[(z^{2/3}/\kappa_0^{2/3})-z^2]^{-1/2}[(1/\kappa_0^{2/3})(2/3)z^{-1/3} - 2z] = (1/2)[1/\rho][(1/\kappa_0^{2/3})(2/3)z^{-1/3} - 2z]$. Then the RH term is zero, and substituting for κ_0 using eq.5.76b gives $(1/(\tilde{\omega}^2 z)^{2/3})(2/3)z^{-1/3} = 2z$. Invert to $((\tilde{\omega}^2 z)^{2/3})(3/2)z^{1/3} = 2/z = (\tilde{\omega}^{4/3})(3/2)z = ((\mu/r^3)^{2/3})(3/2)z = (1/r^2)(3/2)z$ so that $(3)z^2 = r^2 = (\rho^2 + z^2)$ and finally $\rho^2 = 2z^2$, eq.5.78. Amazing and convoluted.
Try eq.5.76 in the form $\kappa = \tilde{\omega}^2 z = z/(\rho^2 + z^2)^{3/2}$ and look at $d\kappa/d\rho = 0$.

p202.39 "not in fact lie along (Sol) line." Useful for Pole communications.

p202.40 Change "off axis" to "off sol-planet axis" or something similar. For some reason I noted that $\rho\bullet\kappa = \cos\varphi$.

p202.44 "along the +z axis over the planetary day side." In fig.5.17, z-axis has an arrowhead, but z on the **k** radius is a distance. Needs development and better figure.

p203.21 Fig.5.16 To caption add "acceleration κ contours". C_3 defined p202.33. C_4 defined p.206.34. Acceleration contours *κ*. Add more to graph to Solward of planet.

p203.35 Fig.5.17. z is both Sol-planet axis and a vector at angle ϕ to z-axis. Blah. Need such orbit with $\varphi = 66.5°$ and $T = 24$ hours so sail is fixed relative to Pole and CONUS. Or is that "fixed" meaning only one degree of motion? ***1*** is different font than text.

p203.44 "perturbation **δ** such that $\mathbf{r}_0 \rightarrow \mathbf{r}_0 + \boldsymbol{\delta}$" see p207b.

p204.10 For ξ' and η' see p182.15.

p204.12 "drift along the nominal orbit" means trouble for synchronous orbits as in note for p203m, but see sec.5.3.4. After that I get lost again.

p205.14 and .20 Curved < and > signs should just be < and >.
Eq.5.86 and eq.5.87 derive from eqs.5.85ab.

p205.23 The stable regions in fig.5.14 are the upper left and lower left corners.
Add "ρ" and "z" to "large radius ρ and small displacement z".

p205.25 "$\rho = (2/3)^{1/3}r_0$" ≈ $0.874r_0$. With $r_0 = 30$ the intersection is at $\rho = 26.2$.

p205.42 Set RHS of eq.5.88b > 0.

p206.09 and .13t For eqs.5.91 and .92, follow book's substitutions rather than solve eq.5.90 for z/ρ. Eq.5.92 $\gamma \equiv (1+\varepsilon^2)^{1/2} \Rightarrow \varepsilon^2 = \gamma^2 - 1$.

p206.32 For eq.5.95 remember that $r^2 = \rho^2 + z^2$.

p206.35 The stable regions in fig.5.16 are to the left of C_4.

p207.19 Fig.5.18 Show planet in half-lite from left. ξ_0 and η_0 are displacements.

p207.30 Section 5.3.4 loses me.

p207.36 For ξ' and η' see p182.15.

p208.01 Using **K** for both pitch control and area control is confusing. Differentiate with eq.5.97a $\mathbf{K}^\alpha$ and eq.5.97b $\mathbf{K}^\kappa$. Similarly, the components are
eq.5.98a $\mathbf{K}_1^\alpha$
eq.5.98b $\mathbf{K}_2^\alpha$
eq.5.99a $\mathbf{K}_1^\kappa$
eq.5.99b $\mathbf{K}_2^\kappa$

a_ρ, a_z are accelerations in **ρ^** and **z^**.

p209.18 Section 5.3.4.2 Acceleration κ will be controlled by varying the sail area, or the reflectivity of the sail as was done by IKAROS. The pitch angle α will be constant at zero in hwat follows.

p209.34 x_j form state vector? Hwat are they explicitly? I am lost.

p210.18 Fig.5.19. Sun to left along –z-axis. Need half-light on planet, label on x-axis. And a diagram for axis not along Sol line, as p203m.

p210.44 Fig.5.20 Excursions below zero look like they diverge. Hwat is maximum excursion +/-?

p211.01 A change in the small continuous (not necessarily constant) acceleration of a sailcraft or ion engine is called a patch, in contrast to the matching of orbits by a short but relatively large impulse, as from a rocket.

p211.19 A Keplerian orbit is partly described by the *vis viva* equation, $v^2 = 2\mu/r - \mu/a$, with v the speed in any direction, $\mu = GM_{\text{Central Body}}$, r = distance between central body and space craft, and a the semi-major axis of the resulting orbit (here typically an ellipse). The non-Keplerian orbit is a circle of radius ρ and orbit rate ω, so v = ωρ. The non-Keplerian orbit is offset from the central body by z, and $r = \sqrt{\rho^2 + z^2}$. Making the substitutions and re-arranging the *vis viva* equation results in eq.5.106.

p211.23 "apocentre" is generic for apogee, aphelion, et cetera. It is "apo" because a sail in circular orbit moves more slowly than the corresponding Keplerian orbit, since the effective gravity of the central source is smaller.

p211.24 Eq.5.107 As the sail goes edge-on to Sol, it will begin an ellipse at the apocentre $r = \sqrt{\rho^2 + z^2} = r_A$.   For an ellipse $a(1+e) = r_{\text{aphelion}}$.

p211.28 Eliminate a from eqs.106 and 107 to get the eccentricity in eq.108.

p211.33 Eq.5.109 For a circle $e = 0$. Put 0 into eq.5.108, solve for ω.

p211.33 "transfers from off-axis orbits" Elaborate. This is the Sol-planet axis.

p212.15 Fig.5.21. Sol to left. All three orbits are circular. Orbit II-upper and Orbits II-lower are centered on the z-axis. Any point on all three orbits are the same distance r from the planet, easiest to see at the tangents to Orbit I. Label the Orbit I axis with **k**, per fig.5.17 p203. Redraw with **k-** and z-axis in plane of page. In book, Orbit II-upper does not pass thru the Orbit I/Orbit II-lower contact, just bad perspective.

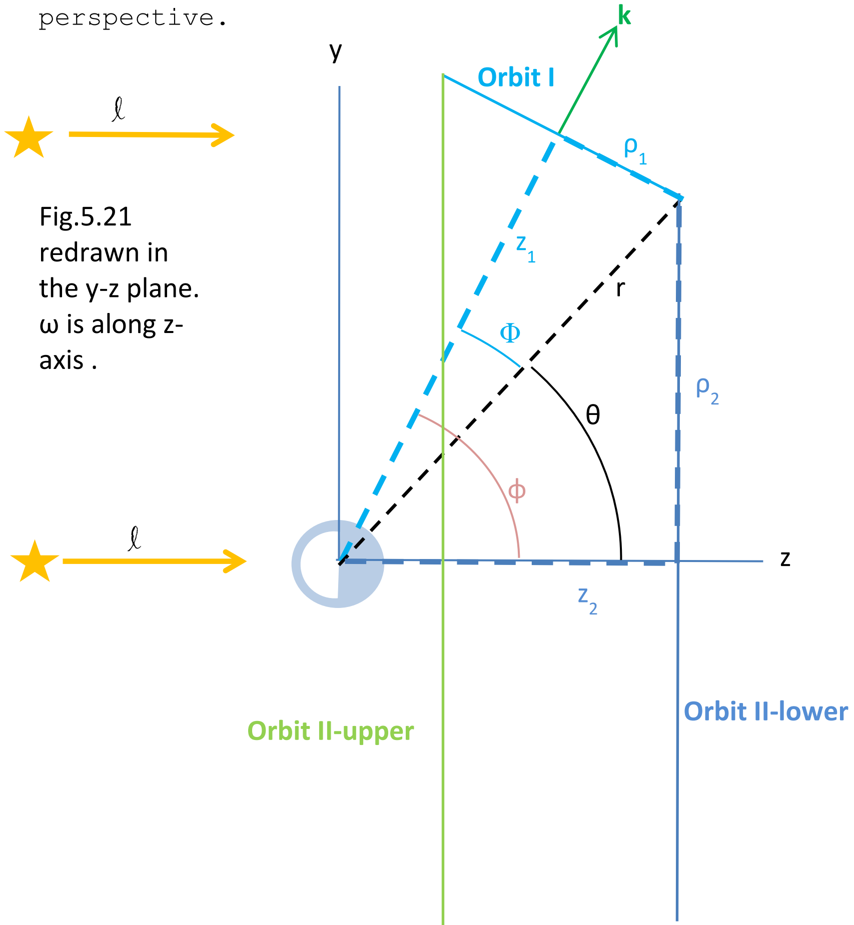


Fig.5.21 redrawn in the y-z plane. ω is along z-axis .

All orbits are shown edge-on.

$\tan\Phi = \rho_1/z_1$
$\tan(\Phi - \phi) = \rho_2/z_2 = \tan\theta$
$r^2 = \rho_1^2 + z_1^2 = \rho_2^2 + z_2^2$
Orbit II-upper and Orbit II-lower are centered on z-axis and tangent to Orbit I in y-z plane.
Angular speed of Orbit I is ω = ω~.
Angular speed of Orbit II-lower is ω_1.
Angular speed of Orbit II-upper is ω_2.

p212.17 "The off-axis Orbit I is an "

p212.21 Eq.5.10a uses increase factor of $(\mathbf{l}\cdot\boldsymbol{\kappa})^{-2} = (\cos\phi)^2$ from p202.42.

p212.28 eq.5.111 $\rho_1\omega\sim = \rho_2\omega_2$. The subscript "2" can refer to upper or lower Orbit II.

p212.42 Eq.5.113 lacks $(1/\cos^2\varphi)$ in rightmost term per p202.42. It comes from $\kappa_2 - \kappa_1$, consistent with signs of eq.5.113. Insert +Φ for Orbit II lower and −Φ for Orbit II upper.

p212.44 Check that fig.5.22 is consistent with eq.5.113 with the factor $(\cos\phi)^{-2}$.

p213.29 Fig.5.22, in caption define T_1 from Orbit II lower and T_2 from Orbit II upper. The point of the line $\varphi + \Phi = \pi/2$ is that it is the limit that keeps Orbit II on the nite side.

p213.31 "transfer back from Orbit II (lower}" , with z_1 rotated about z relative to its original position.

p213.39 "keep Orbit II on planetary night-side." => $\varphi + \Phi = \pi/2$

p213.44 "composed of small" need example.

p214.09 Hwere are the "four new additional equilibria"?

p214.39 $\mu = m_2/(m_1 + m_2)$ defined, not μ = MG as earlier.

p215.17 Fig.5.23 Mark center of mass at axes intersection. Show $\boldsymbol{\omega}$ coming out of page, centered at origin. Triangles are equilateral. If m_1 is luminous there are two more points L_6 and L_7, locations unknown.

p215.24 " is ~~modified~~ reduced "

p215.38 Remember that **n** is fixed after this, and **ω** = 1.

p216.11 Fig.5.24 Make ω bold **ω**. m_1 is Sol = luminous body, make open circle. Axes cross at center of mass. Label m_1 at $-\mu$, m_2 at $1-\mu$. Sail is not in x-z plane.

p216.18 Eq.5.114 same as eq.5.64. Eqs.5.117ab same as eqs.5.66ab.

p216.34 Add "/" to "$m_2/(m_1+m_2)$.

p217.01 "must vanish, so that del**U** = a."

p217.15 cone and clock angle defined. Hard to visualize.

p218.24 Fig.5.25 Since this now represents the solar system, m_1 represents luminous Sol and essentially sits on the co-ordinate system origin, not at $-\mu$. In (c), m_1 is to left. Hwat does the area around m_2 look like in the y-z plane?

p219.10 $L_1 < x < 1-\mu = m_2 \approx 1$ since m_1 is Sol and $\mu \approx 0$. From p214b $\mu = m_2/(m_1+m_2)$

p219.15 $\beta \rightarrow \infty$, see fig.5.26 p220t.

p219.23 "single sail attitude angle" I think this is α.

p219.39 Should read "that ~~their~~ there are surfaces around".

p220.22 Add "forbidden" hash marks to graphs.
Caption fig.5.26, add "on p177" to "(see Table 5.1 type 1 on p177 for values)".

p221.09 How to derive eq.5.126, and its predecessors. Then I get lost anyway.

p225.16 Offset angle in fig.2.7 p47.

p225.35 as used here, in fig.2.7, **u** // **r**, I think.

p226.35 Add "ness" to "a lightness number".

p227.20 Fig.5.28 To caption add "reflectivity 0.9", "dashed lines" and "solid lines" Add Sol and Terra to drawing. Compare to first quadrant of fig.5.2 p176.